Mon. Apr. 02, 2018

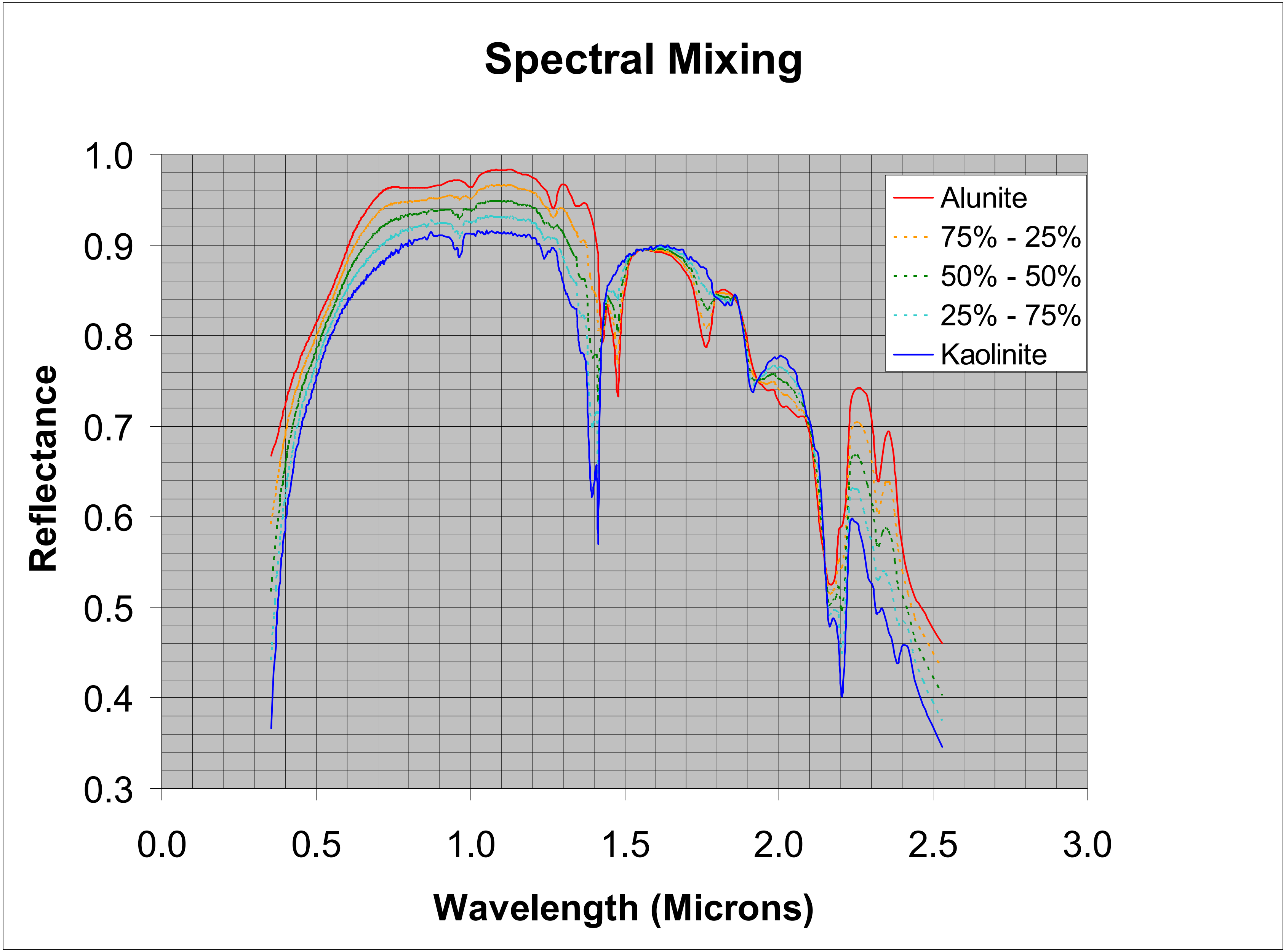
- Satellite Gravity Measurements (GRACE, GRAIL)
  - Use slides posted for Friday
- Linear (Spectral) Mixing

- Reading: Chapter 9 ("Environmental" Remote Sensing")
  - Once again -- Satellites old but principles still apply

"Linear Spectral Unmixing 1"

- Linear: Assume resulting spectra is weighted average of endmember spectra.
  - (Only true for macroscopic mixing. Microscopic (intimate) mixing is often non-linear.
- Following is true for every pixel. Repeat mathematics for each pixel.
- Observe some spectrum $I(\lambda)$
- There are N possible minerals present, each occupying a fraction f_j of the pixel (for $j=1, N$)
  - Usually you assume $\sum_{j=1,N} f_j = 1$ i.e. pixel is fully occupied
- You know the spectrum $R_j(\lambda)$ for each of the j=1,N minerals
- For linear mixing $$I(\lambda) = \sum_{j=1,N} f_j R_j(\lambda)$$
- **Given $I(\lambda)$ and $R(\lambda)$, find the fractions f_j**
- ***Complications***
  - ***Noise***
  - ***Observations only at a limited number of λ***

Mineral Spectra: Example of linear mixing



"Linear Spectral Unmixing 2"

- Simple example: Single wavelength, and observed $I_1 = 0.5$
  Two minerals, with $R_{1\,1} = 0.4$ and $R_{1\,2} = 0.7$ (I'll explain the first R subscript shortly)

$$
\begin{aligned}
I_1 &= R_{1\,1} f_1 + R_{1\,2} f_2 \\
&= R_{1\,1} f_1 + R_{1\,2} (1 - f_1) \\
0.5 &= 0.4 f_1 + 0.7 (1 - f_1) \\
0.5 &= -0.3 f_1 + 0.7 \\
-0.3 f_1 &= -0.2 \\
f_1 &= 0.67 \quad \text{and } f_2 = 1 - 0.67 = 0.335
\end{aligned}
$$

- However if we do not have endmember minerals whose reflectances bracket the observed value, you get "nonsense" results with $f_i < 0$ or $f_i > 1$
  Single wavelength, and observed $I_1 = 0.5$
  Two minerals, with $R_{1\,1} = 0.4$ and $R_{1\,2} = 0.3$

$$
\begin{aligned}
0.5 &= 0.4 f_1 + 0.3 (1 - f_1) \\
0.5 &= 0.1 f_1 + 0.3 \\
0.1 f_1 &= 0.2 \\
f_1 &= 2.0 \quad \text{and } f_1 = 1 - 2.0 = -1.0
\end{aligned}
$$

- More sophisticated versions of "unmixing" can enforce constraints $0 \leq f_j \leq 1$ at expense of "imperfect" fits to the observed reflectance.
- Other versions relax requirement that $f_1 + f_2 = 1$
  and (when you have many wavelengths and pixels) allow presence of an unknown component whose spectrum is similar to the <u>average</u> spectrum of all the pixels.

"Linear Spectral Unmixing 3"

- For linear mixing $$I(\lambda) = \sum_{j=1,N} f_j R_j(\lambda)$$

- If you only have data at discrete wavelength λ_i for i=1,M $$I(\lambda_i) = \sum_{j=1,N} f_j R_j(\lambda_i) \quad \text{or,definining } I_i \equiv I(\lambda_i) \text{ and } R_{i,j} \equiv R_j(\lambda_i)$$ $$I_i = \sum_{j=1,N} f_j R_{i,j} = \sum_{j=1,N} R_{i,j} f_j$$

- Which can be written in matrix form: $$\begin{pmatrix} I_1 \\ I_2 \\ \vdots \\ I_M \end{pmatrix} = \begin{pmatrix} R_{11} & R_{12} & \cdots & R_{1N} \\ R_{21} & R_{22} & \cdots & R_{2N} \\ \vdots & \vdots & \cdots & \vdots \\ R_{M1} & R_{M2} & \cdots & R_{MN} \end{pmatrix} \begin{pmatrix} f_1 \\ f_2 \\ \vdots \\ f_N \end{pmatrix}$$

- Each vertical column in the matrix is the spectrum of a given mineral.

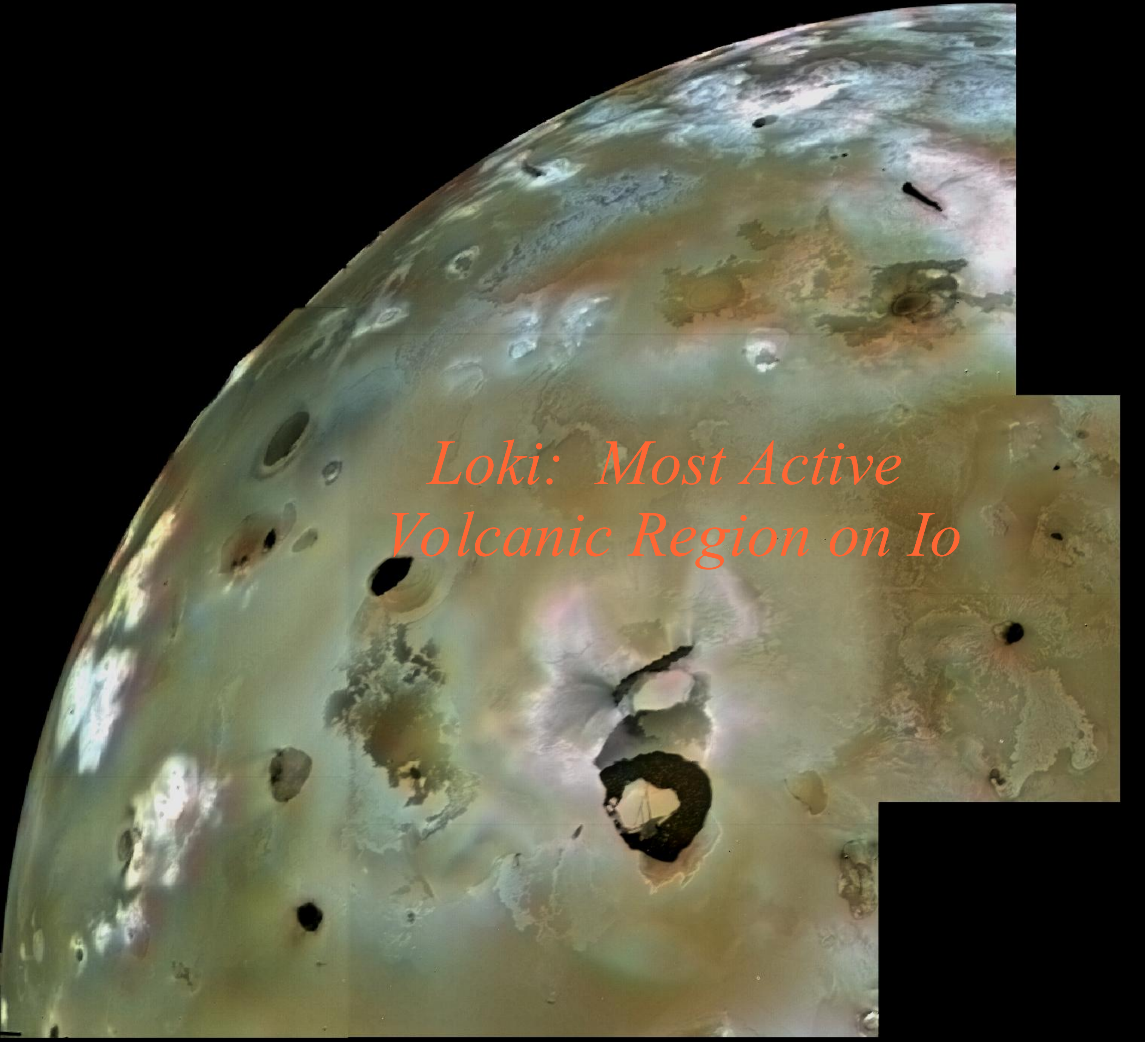
- We want to determine f_i given the known I_j and $R_{i\,j}$
  We have M simultaneous equations (= # λ_i)
  with N unknowns (= # of minerals)

- Including the constraint gives us one more equation. $$\sum_{j=1,N} f_j = 1$$

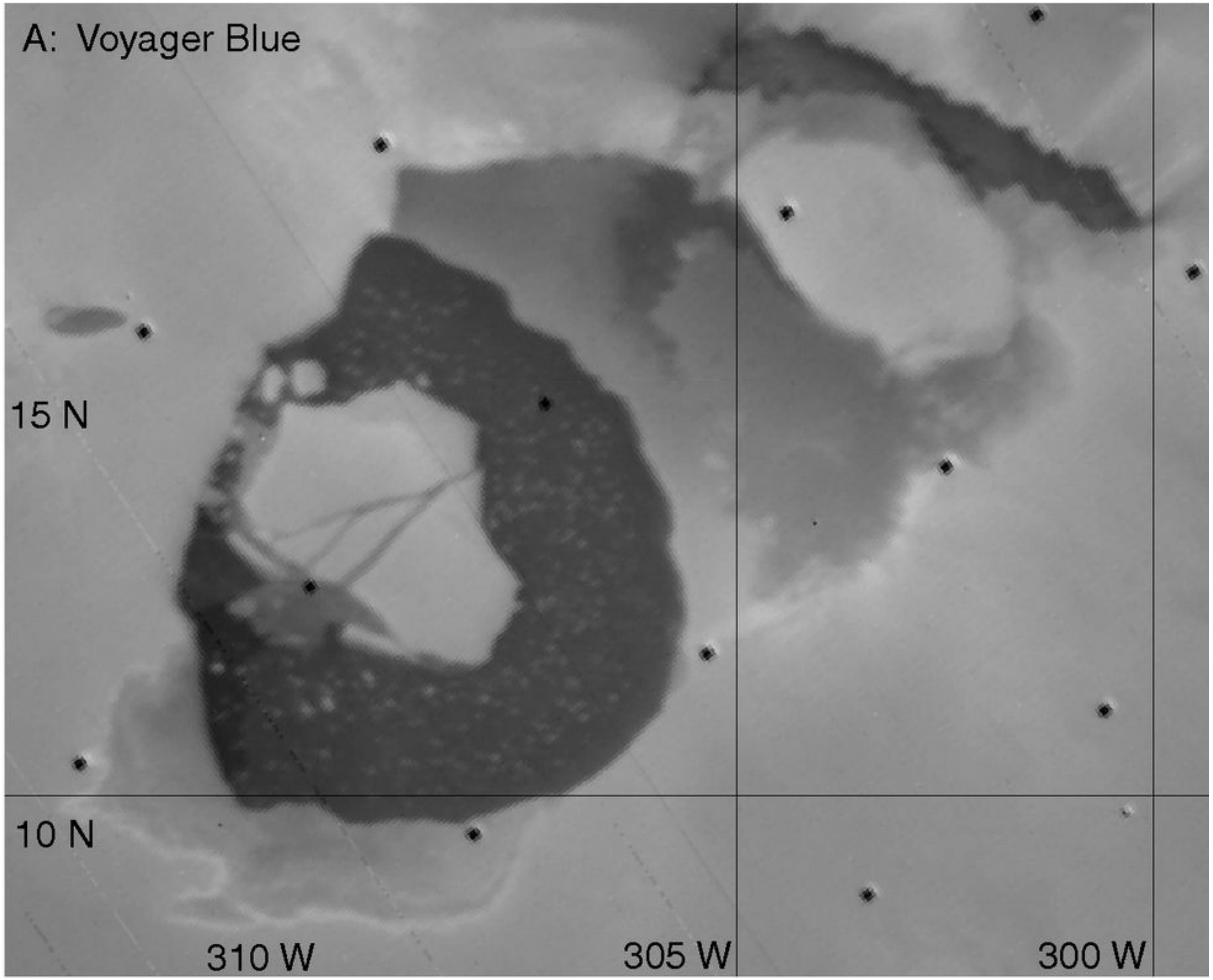
- Since the number of unknowns can at most equal the number of equations, if we have M wavelength bands we can model at most N = M+1 minerals.

- If we have more wavelengths than necessary, the problem is "overdetermined" but we can then produce best fits which in some "average" sense match the observed spectrum "as well as possible".

Linear Mixing Application to Io

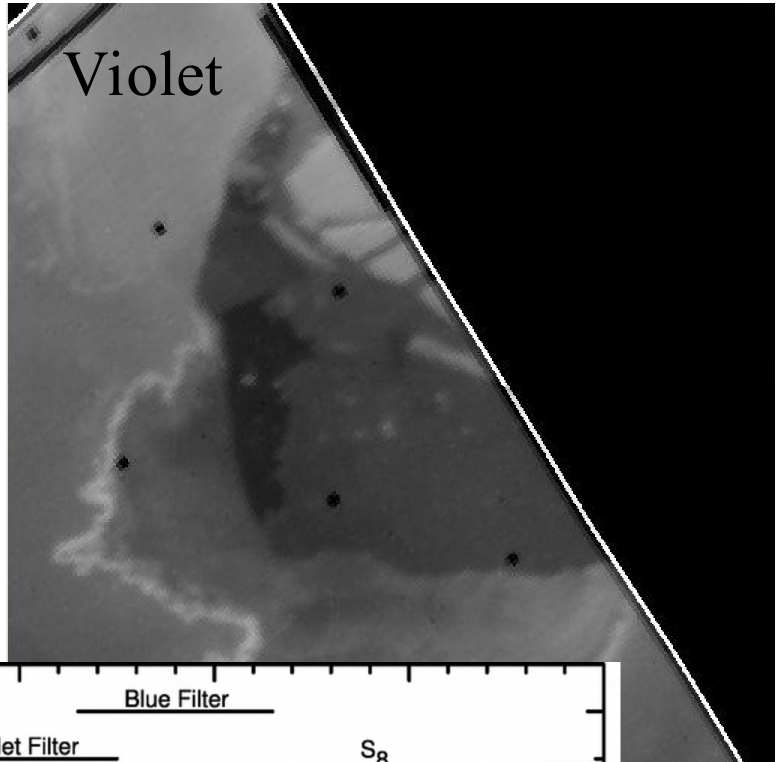


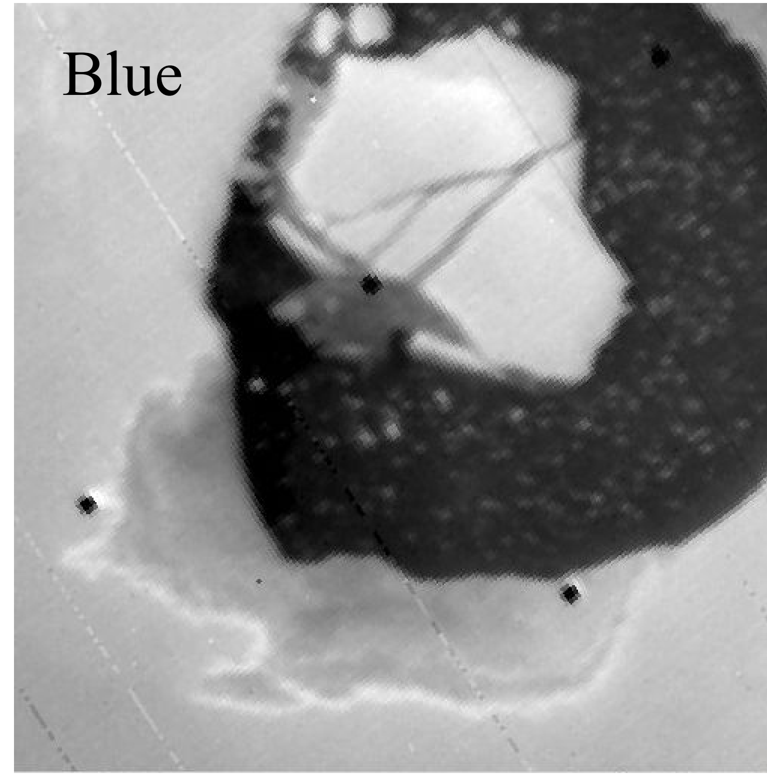
Loki "Bergs": Fumaroles?

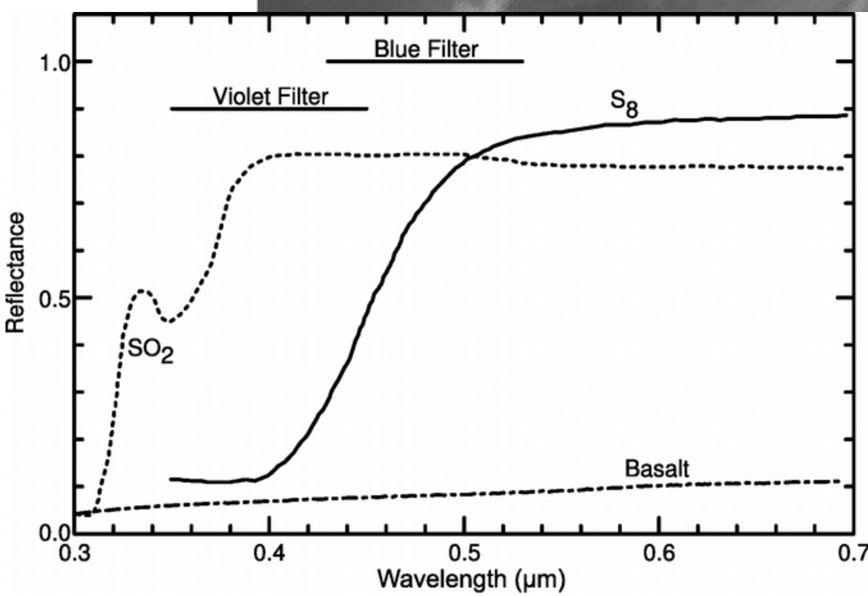


- Thought to be large (200 km diameter) lava lake
- Bright spots may be fumarole deposits
- We're trying to understand their composition

Colors of the “bergs”

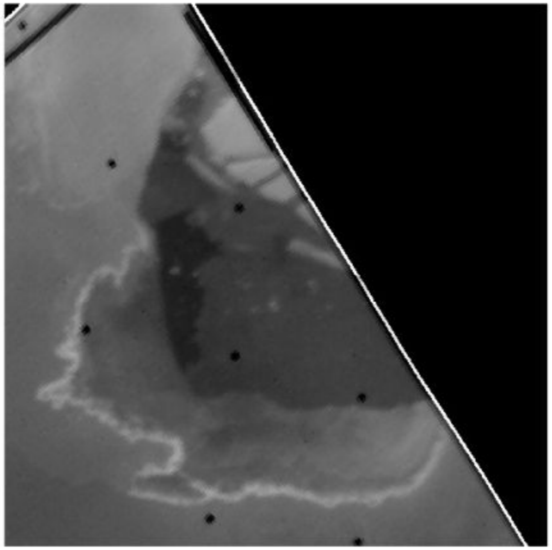




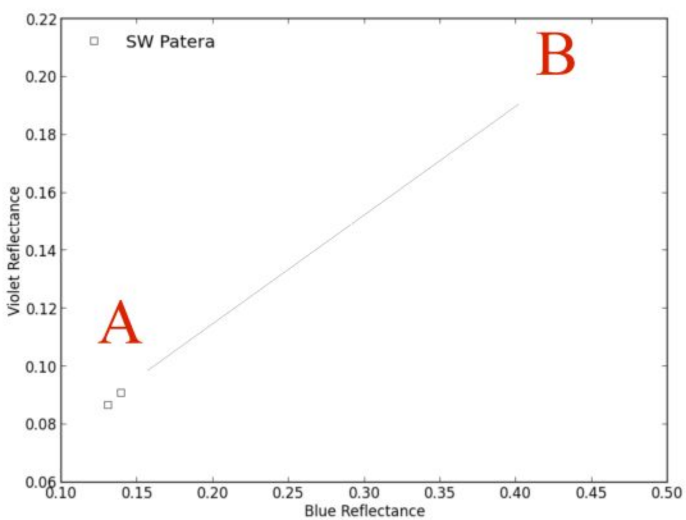


- The “bergs” almost disappear in the violet (The largest are still slightly visible)
- Consistent with a sulfur composition
- Not consistent with (primarily) SO_2 composition
  - Also makes sense in terms of temperatures on patera

Plot Violet Brightness vs. Blue Brightness

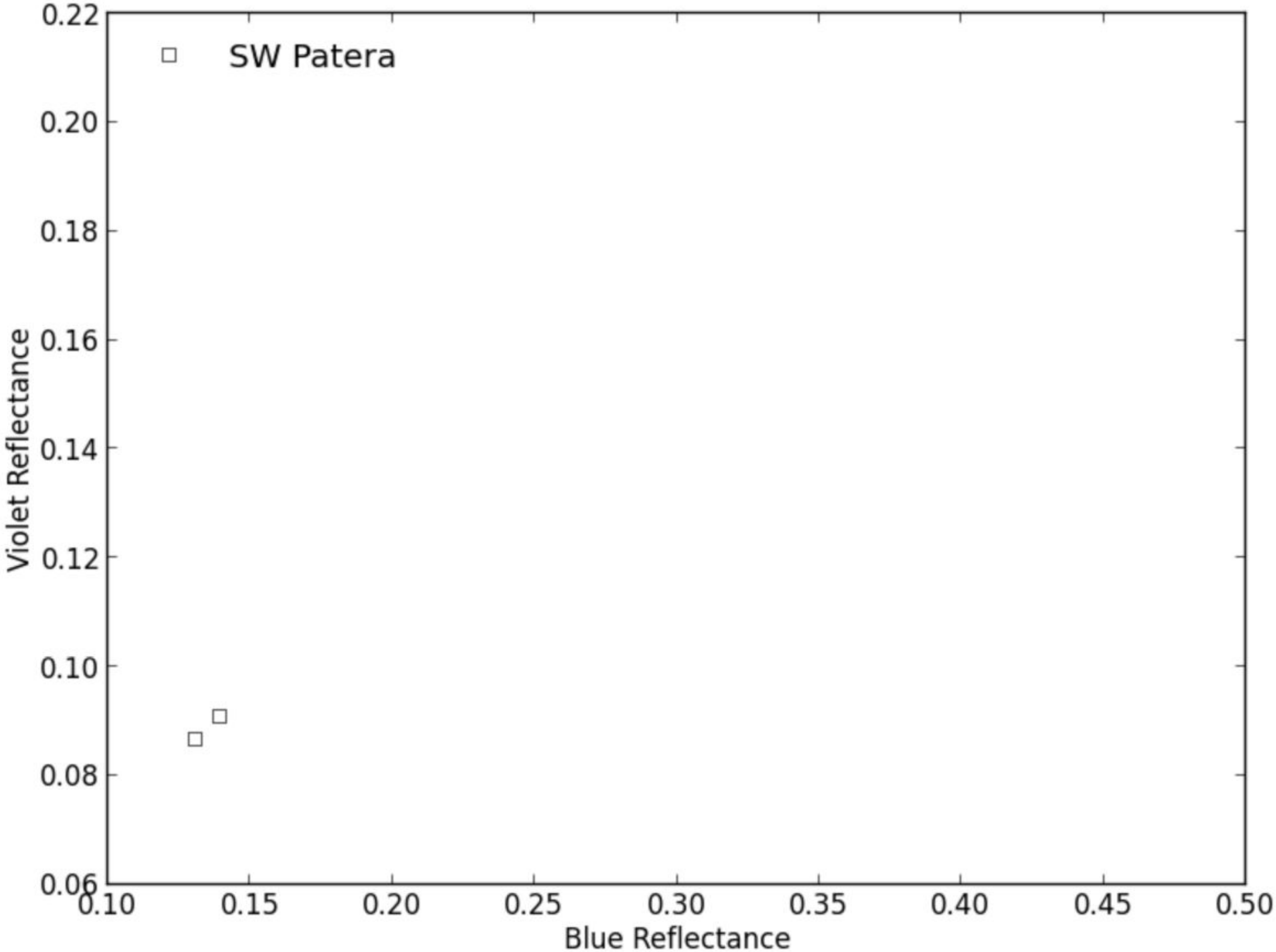


- Imaged reduced and projected to simple cylindrical maps using ISIS3
- Brightnesses are “I/F” values, returned by standard ISIS3 code.
  - While photometric corrections have not been applied, phase angles are relatively small (and similar) and most photometric corrections at these angles are relatively small

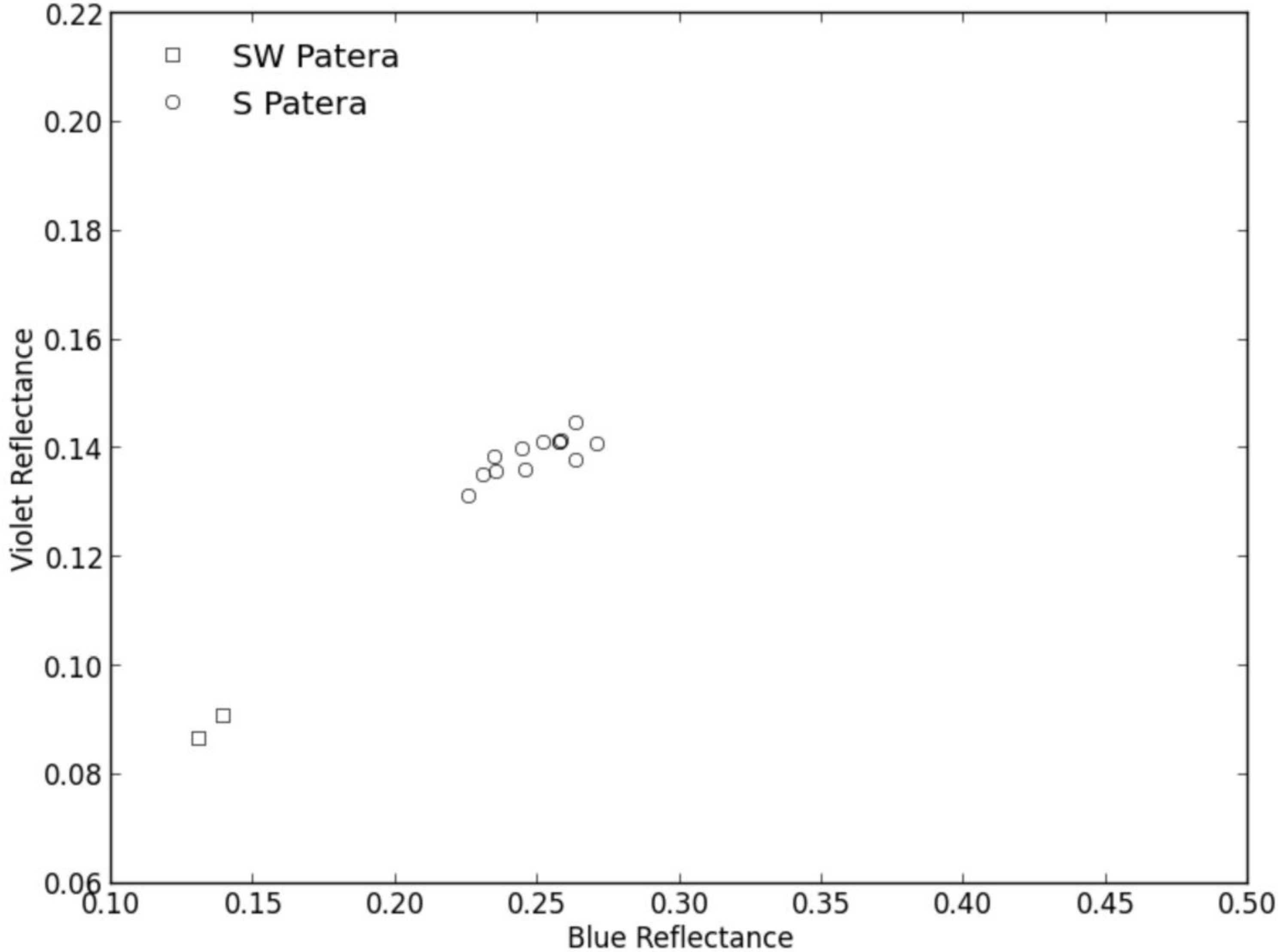


- Linear mixing between endmembers in a two-brightness plot like this simply produces mixing “lines” connecting the endmember reflectance points.

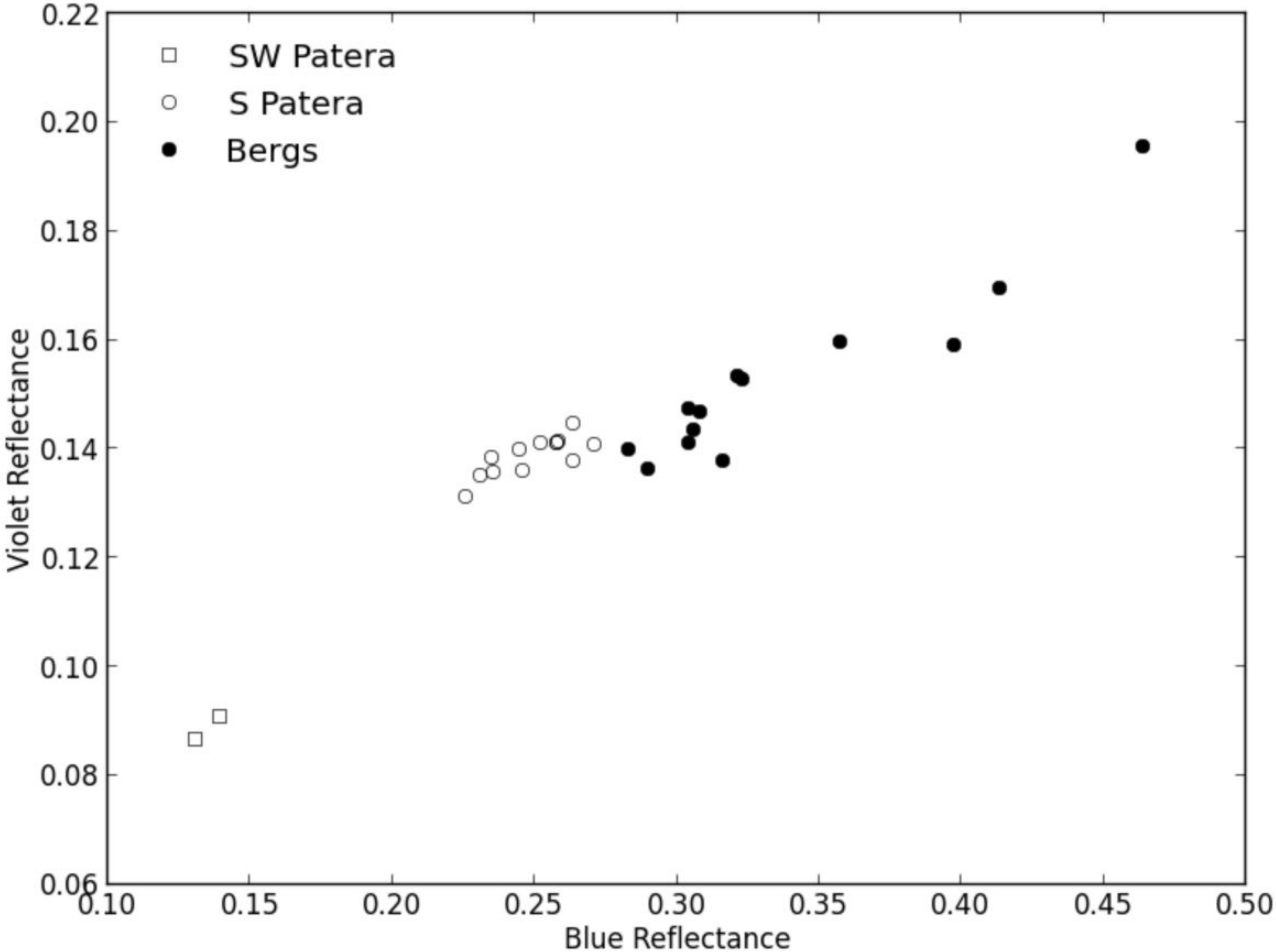
Violet vs. Blue Brightness: SW (dark) Patera Only



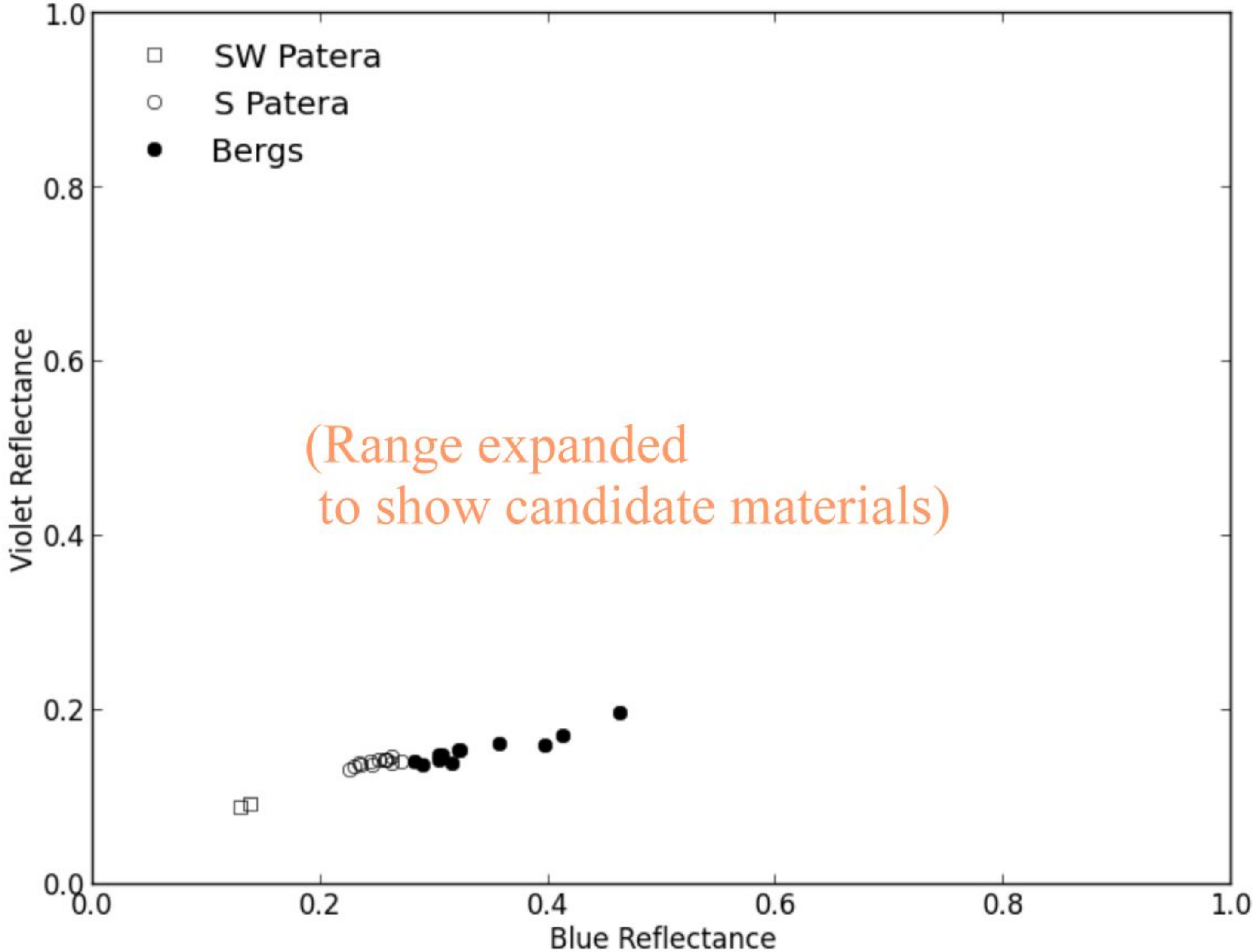
Violet vs. Blue Brightness: SW and S Patera



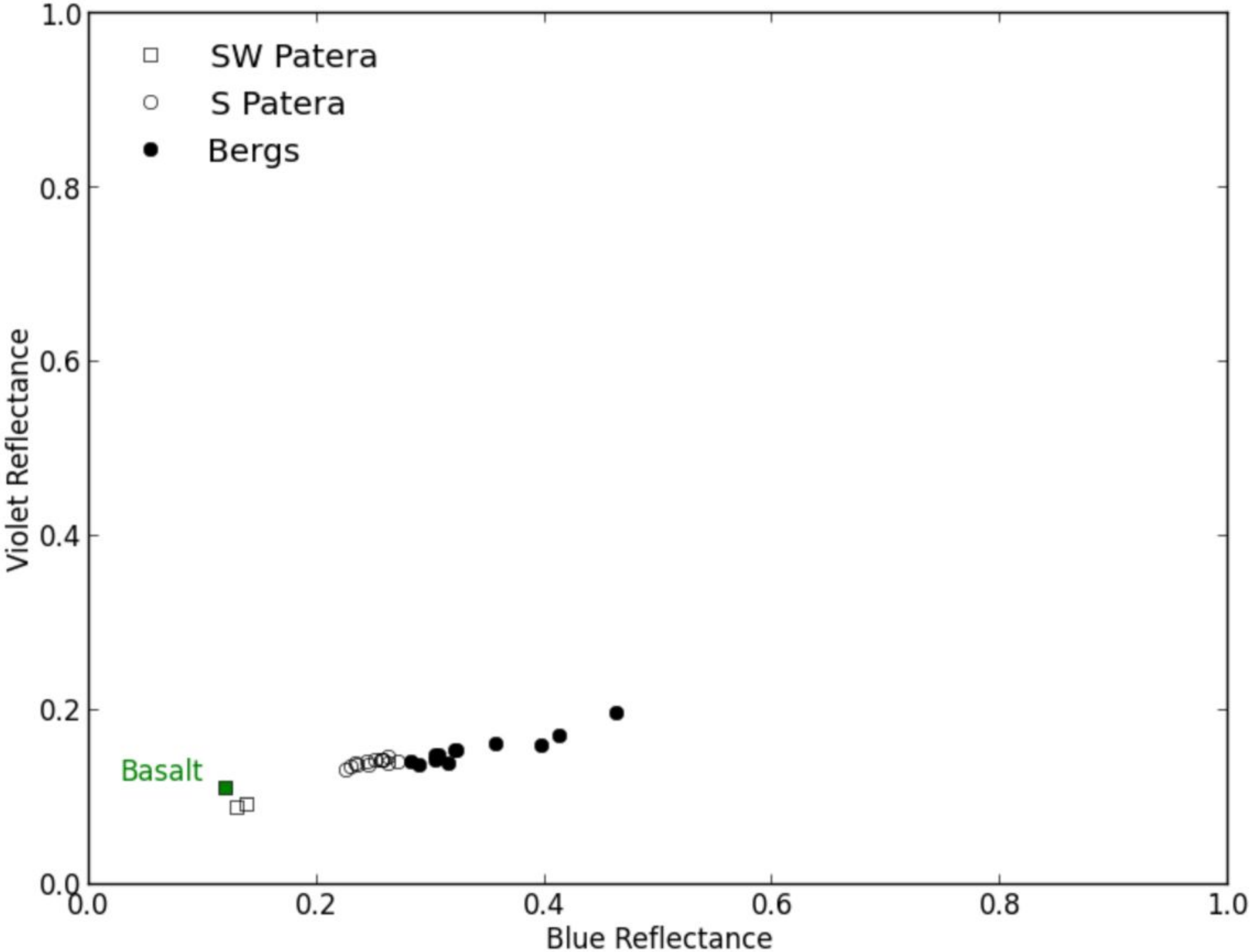
Violet vs. Blue Brightness: SW & S Patera + Bergs



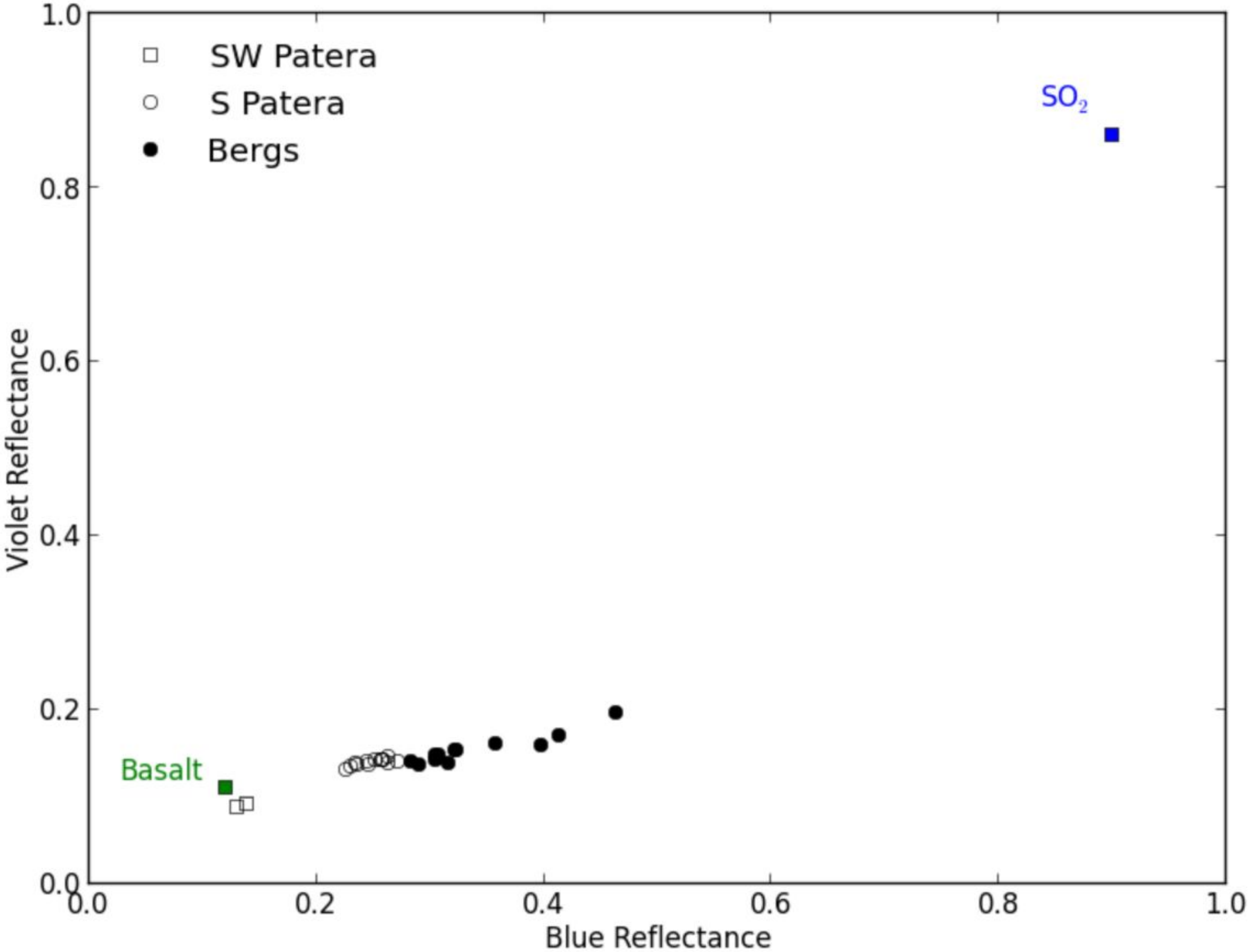
Violet vs. Blue Brightness: SW & S Patera + Bergs



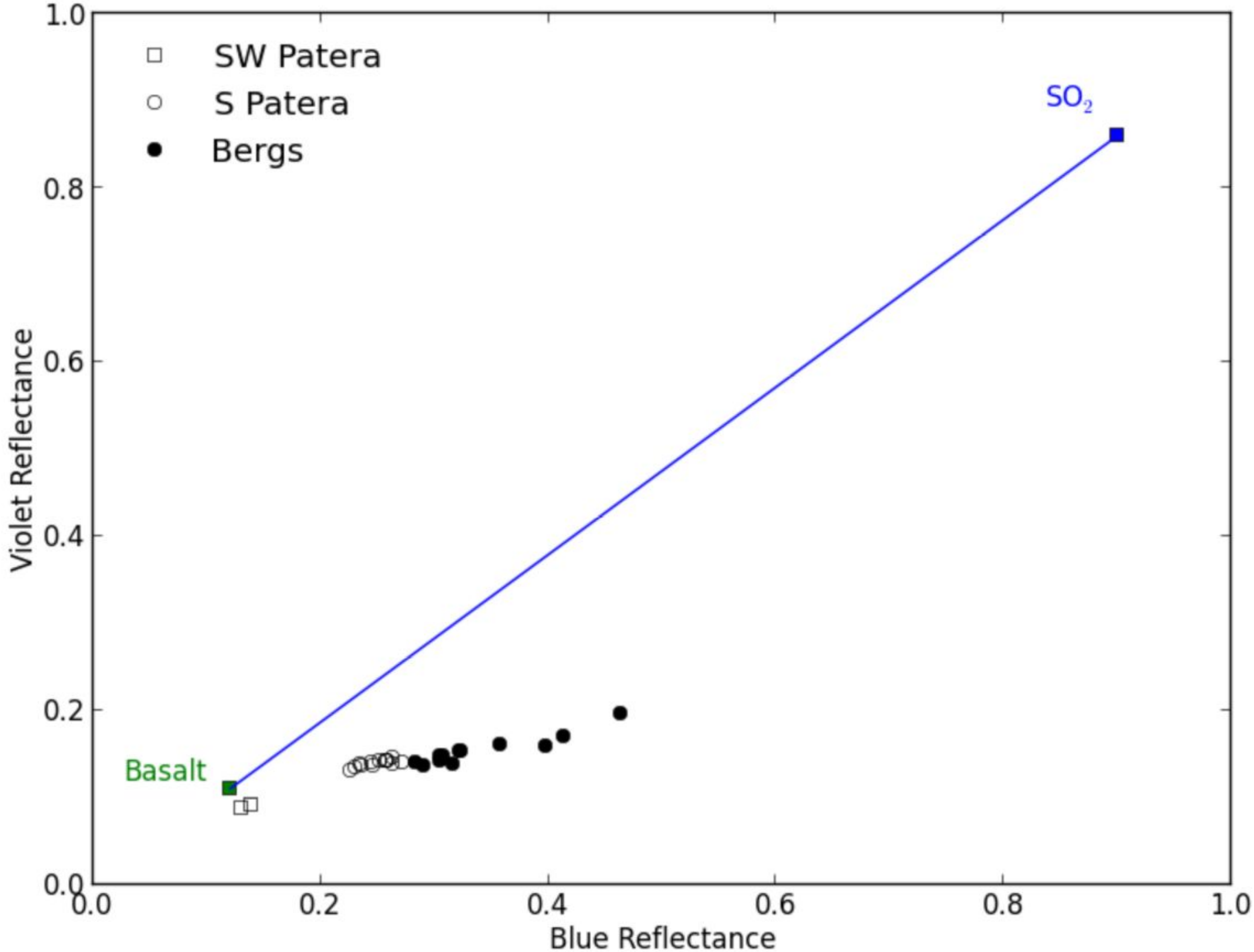
Violet vs. Blue Brightness: SW & S Patera + Bergs



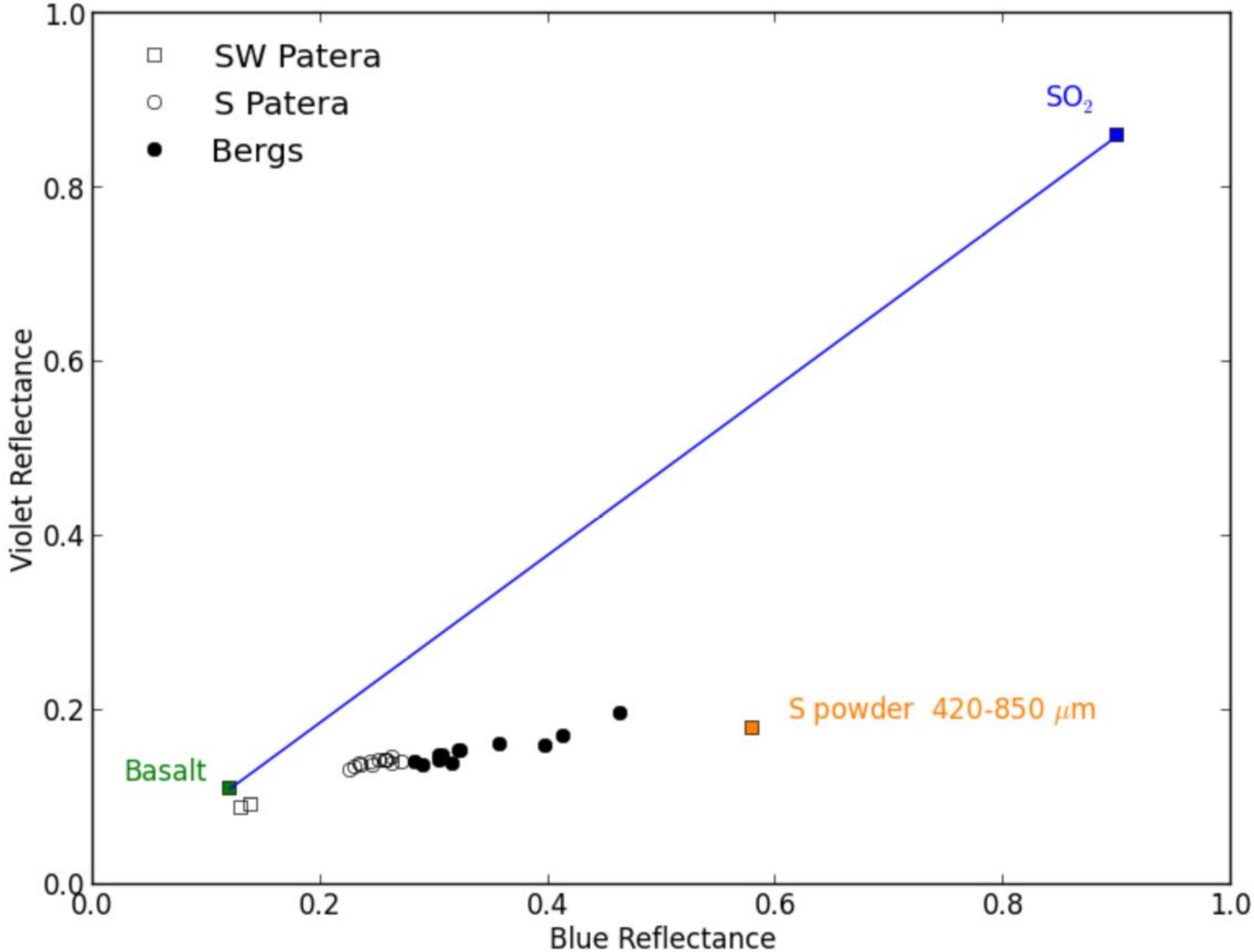
Violet vs. Blue Brightness: SW & S Patera + Bergs



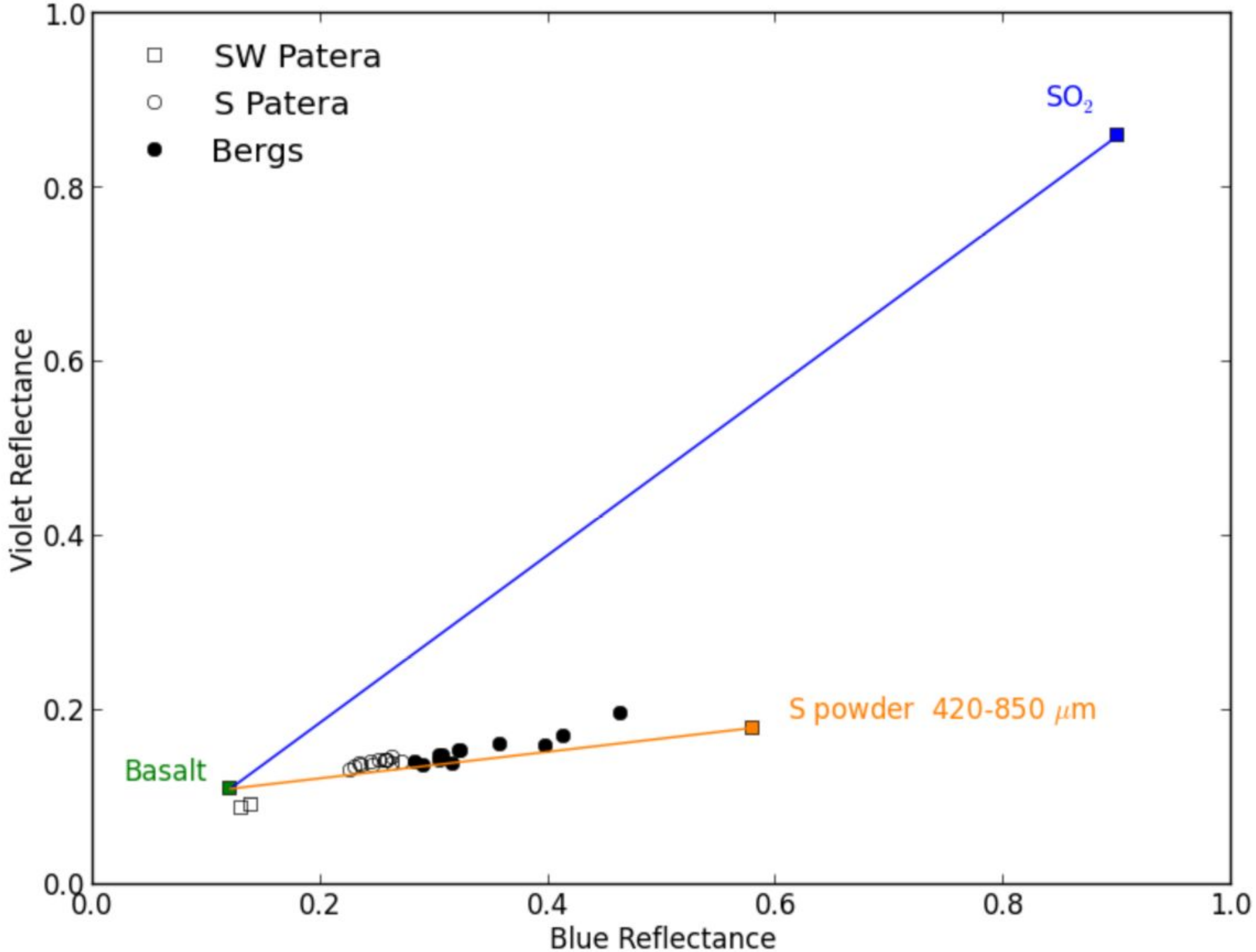
Violet vs. Blue Brightness: SW & S Patera + Bergs



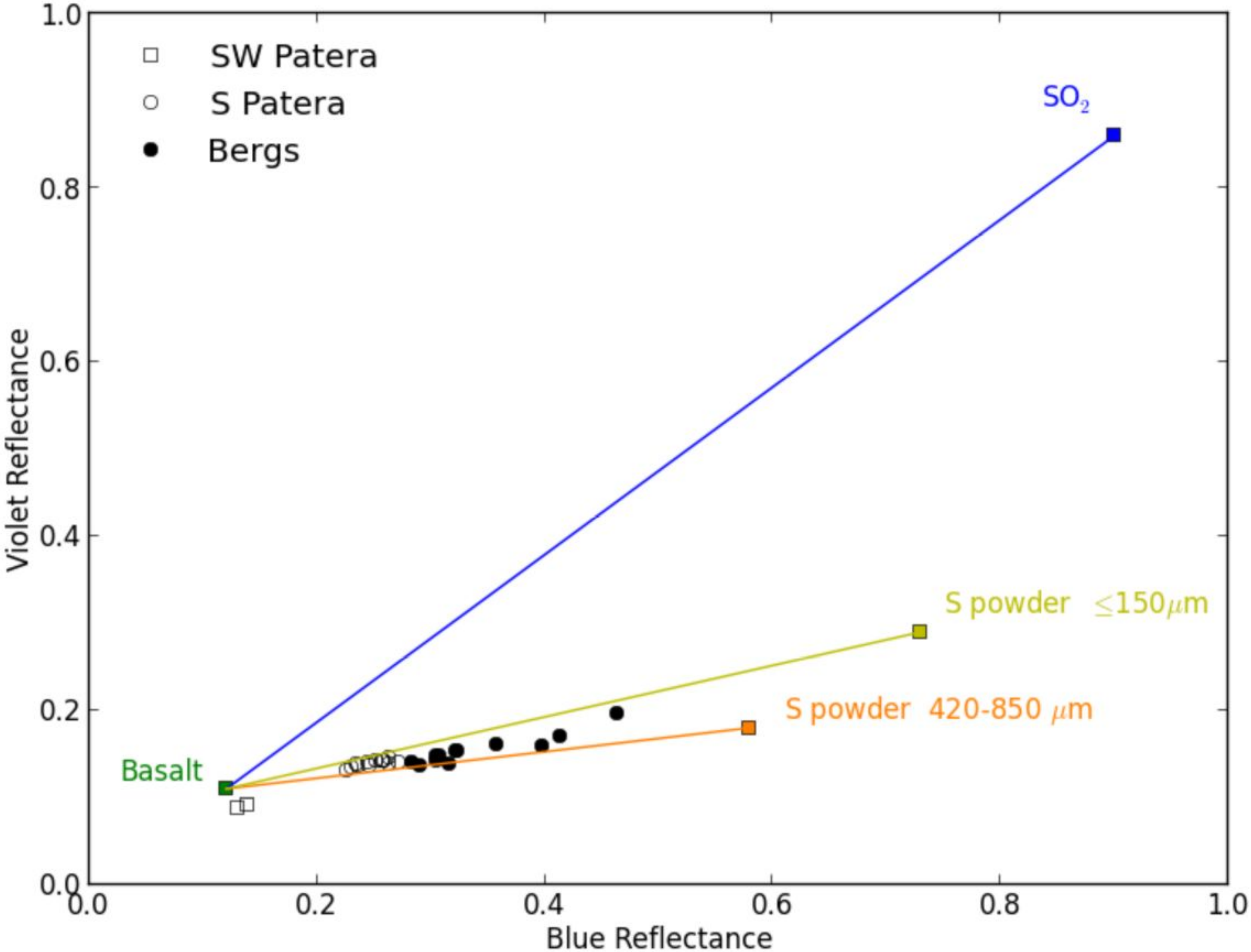
Violet vs. Blue Brightness: SW & S Patera + Bergs



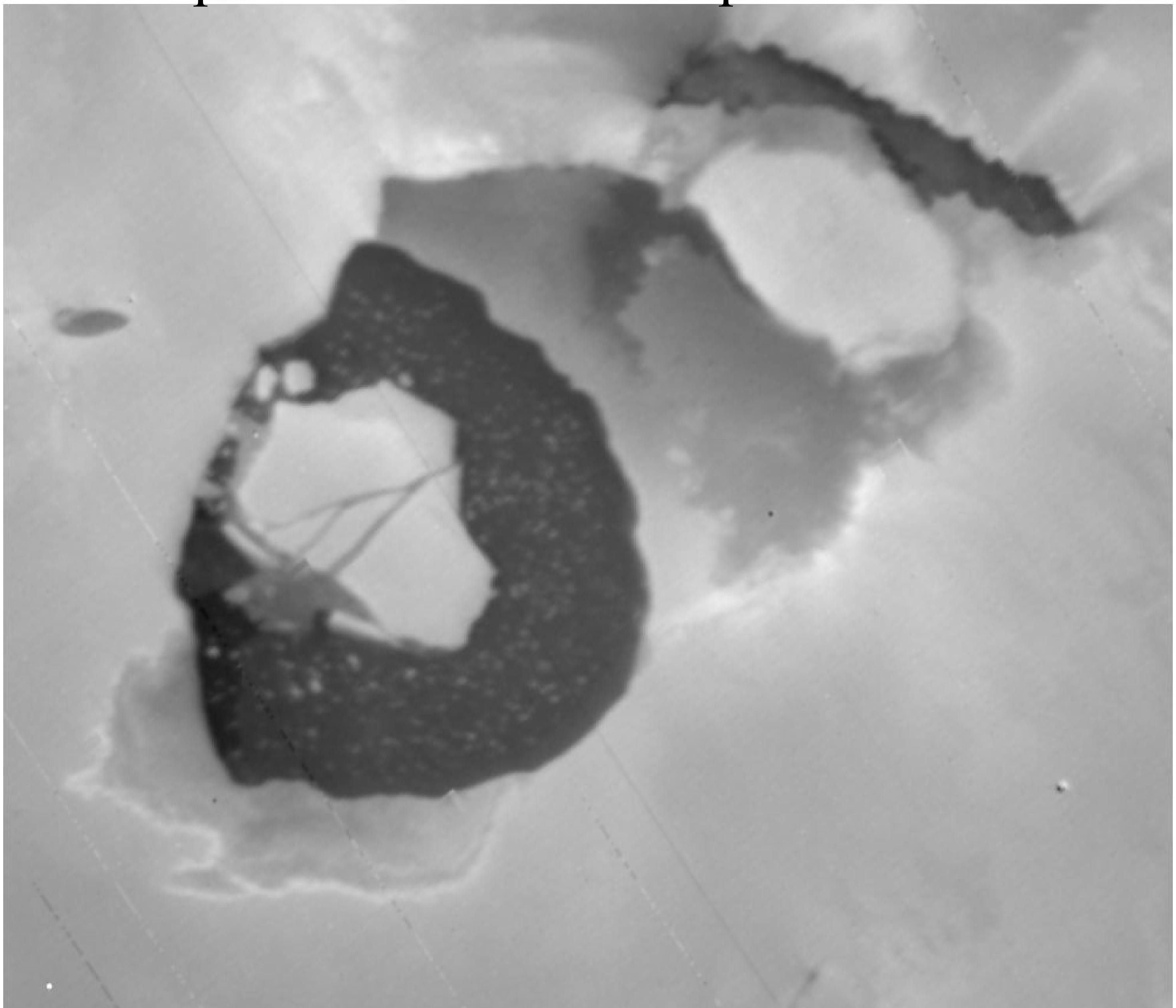
Violet vs. Blue Brightness: SW & S Patera + Bergs



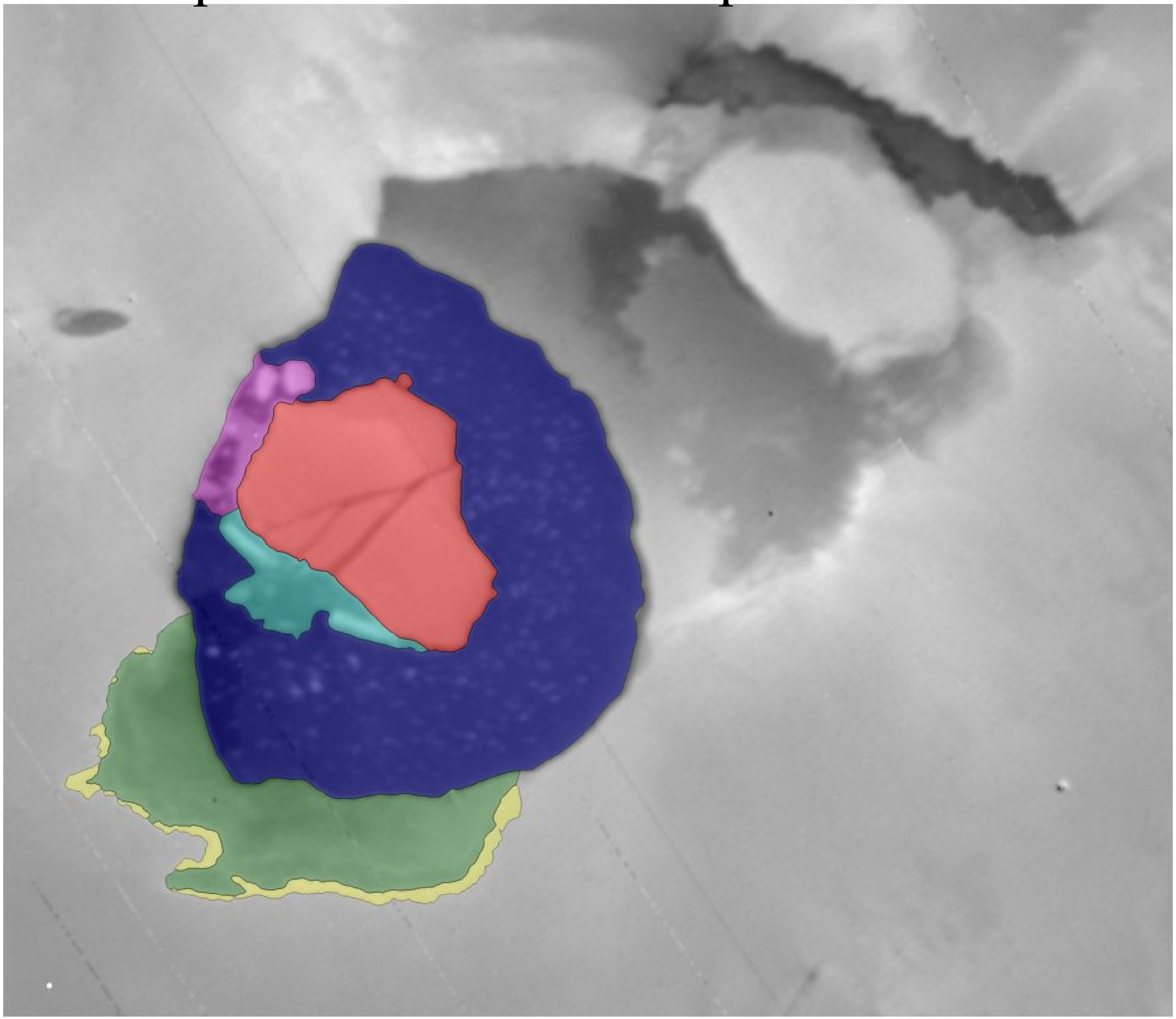
Violet vs. Blue Brightness: SW & S Patera + Bergs

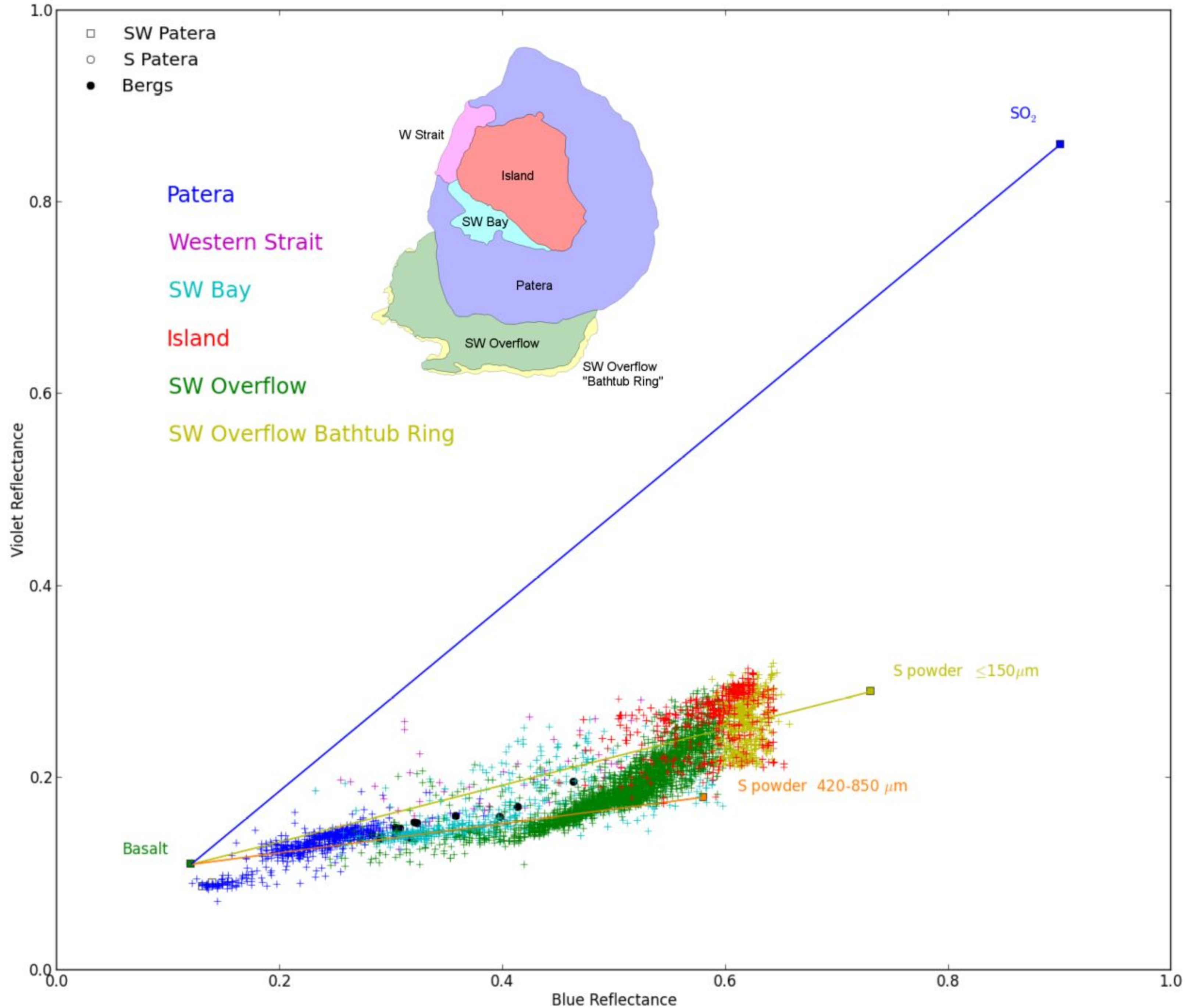


Composition of other Components at Loki?



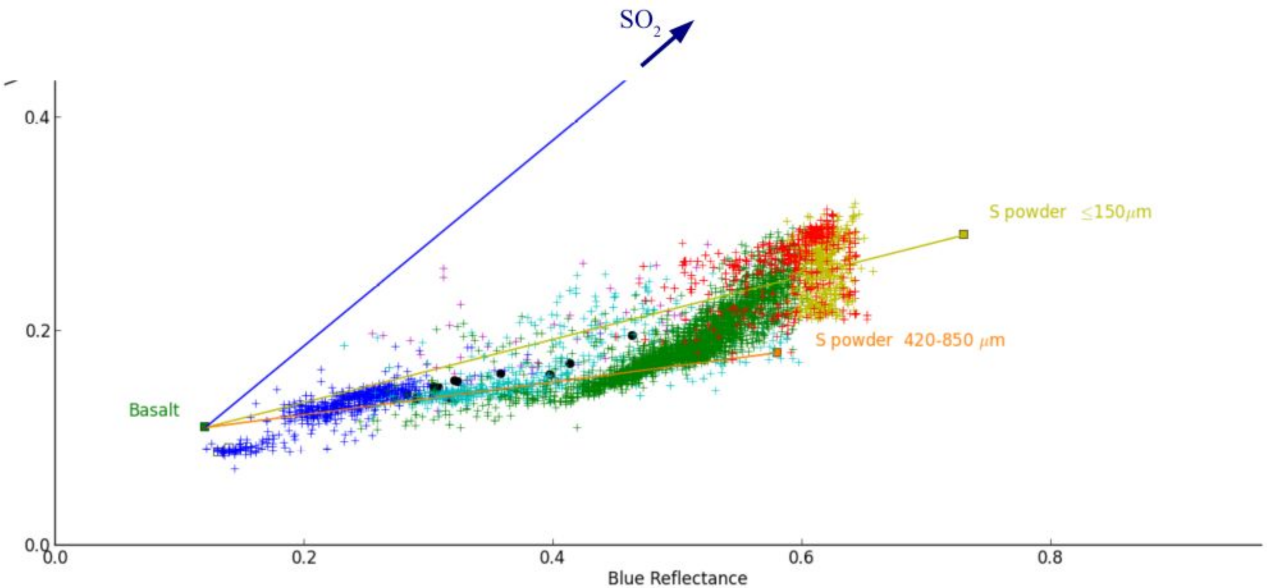
Composition of other Components at Loki?





SW Patera
S Patera
Bergs
Patera
Western Strait
SW Bay
Island
SW Overflow
SW Overflow Bathtub Ring
W Strait
Island
SW Bay
Patera
SW Overflow
SW Overflow "Bathtub Ring"
SO2
S powder ≤150μm
S powder 420-850 μm
Basalt
Violet Reflectance
Blue Reflectance

Composition of other Components at Loki?



1) While the very dark SW corner is likely to be bare basalt -- the slight vertical (V) jump in reflectance between it and the rest of the patera suggests something more than just sulfur deposition affects the violet color. Perhaps this is an age-related "weathering" effect.

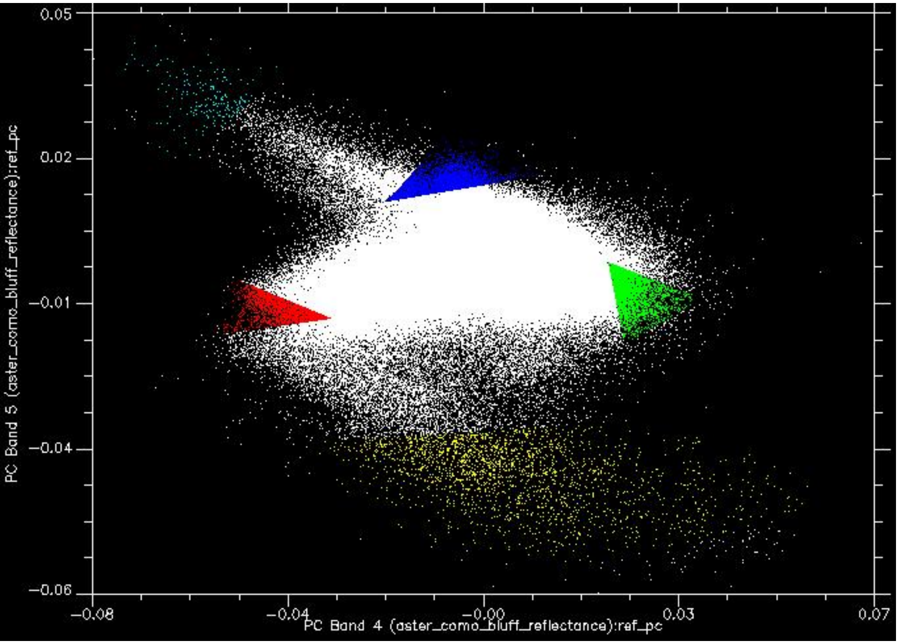
2) The SW Bay and the "overflow region can be explained by basalt, covered by increasing amounts of sulfur.

3) The SW Overflow "Bathtub Ring" and the island COULD be explained by a mixture of more reflective small-grain S -- combined with a dark component like basalt -- but a more natural explanation is:

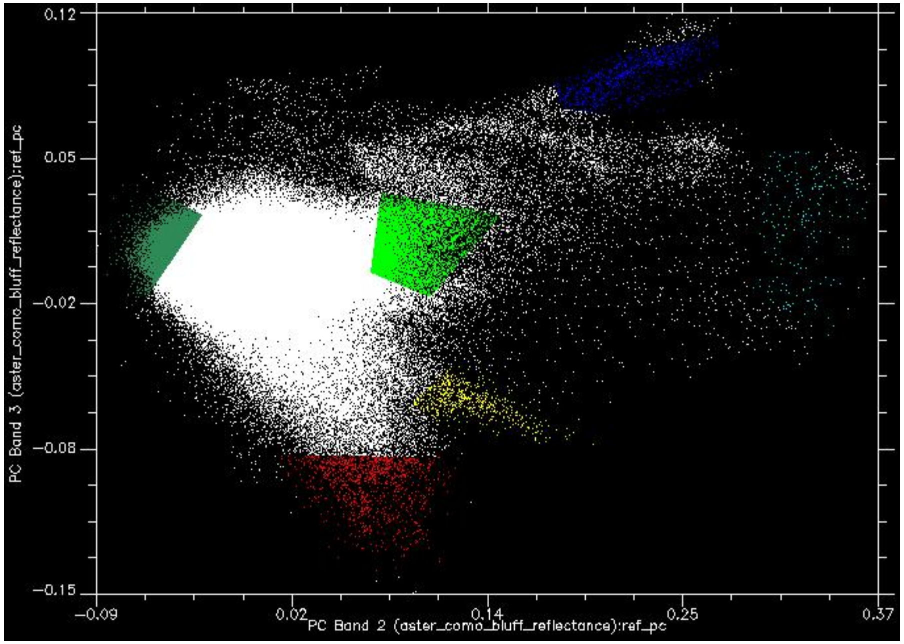
4) Once coverage by course sulfur becomes essentially 100%, the thermal environment may allow condensation of small amounts of SO_2, which then begin to raise the Violet albedo significantly.

Determining "Endmembers" using 2-D Scatter Plots

- Compute PCA or MNP components
- Plot one band of output vs. another
- Possible endmembers are shown by colors in following



PC Band 5 vs. Band 4



PC Band 3 vs. Band 2