Homework #9

**Solution**

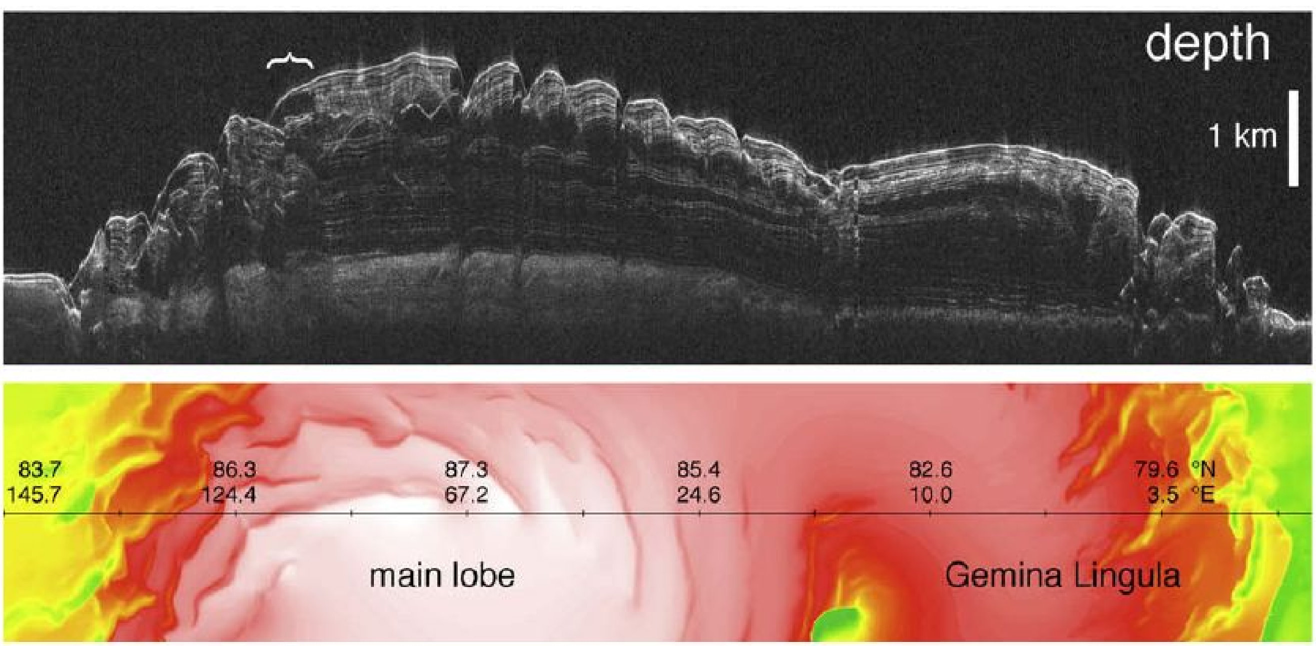
Geology 4113 (Remote Sensing)

Assigned March 30, 2018

Due April 6, 2018

**#1 (35 points total) Ground Penetrating Radar for Mars**
**(and using on-line sources for searching the remote sensing literature)**

The SHARAD (SHAllow RADar) system on the Mars Reconnaissance Orbiter works at a frequency of 15 to 25 MHz where, under the cold Martian conditions, radio waves can penetrate a kilometer or more into ice. The figure below, from Putzig et al. 2009, shows in the lower panel a map of the polar region of Mars and a satellite track across that region, and in the upper panel, the radar reflections seen along that track. The sharp drop in elevation corresponds to the edge of the exposed ice cap (shown a red in the on-line version) and the "basal unit" is the layer that extends horizontally more-or-less continuously from the surrounding non-cap region under the cap. The very top of the ice consists of multiple thin layers which are reflective, because of varying ice/dust ratios and other changing properties. The middle part of the ice is more uniform and therefore radar dark -- since it doesn't present many surfaces to reflect the signal. Under the ice cap the "basal unit" is once again radar bright.




**A) (5 points)** What wavelength (in vacuum) corresponds to the ~20 MHz frequency of SHARAD? (Note this answer is not directly related to the following questions.)

*$c = \lambda v$ or $\lambda = c / v = 3.0\times10^8$ m $s^{-1}$ / $(20\times10^6$ $s^{-1})$ = 15 meters*

*Note this is the wavelength <u>in vacuum</u>. It will actually be smaller in the ice where its speed is slower.*

**B) (10 points)** The radar is aimed straight down. Estimate the maximum thickness of the ice cap from the above diagram, then estimate the (two way) delay time you would expect between the first radar return from the top of the cap and that from the top of the basal unit at the bottom of the cap. In making this estimate, assume that the radar wave propagates at the vacuum speed of light. (In reality it will actually be somewhat slower because of the radar index of refraction of ice, and in processing the data to obtain the above figure the authors needed to compensate for that.)

*From the above image the thickest part of the ice cap is about 2.1 km thick. If you assume the radar wave travels at the vacuum speed of light (which we will correct later) and assume the wave moves vertically, then the reflection from the base unit it must transit an extra 2.1 km twice (going down then up) compared to the reflection from the top surface. Therefore the delay time will be:*

$\Delta t = (\text{two-way-distance}) / c = 2\, \Delta d / c = 2 \times (2.1\ \text{km}) / (3.0\times10^8\ \text{m s}^{-1}) = 1.4\times10^{-5}\ \text{s} = 14$ *microseconds*
where $\Delta d$ is the one-way thickness of the ice cap.

**C) (10 points)** Find the above paper using whatever on-line search techniques you find most convenient. The one I use (for planetary papers) is the NASA Astrophysical Data System (ADS) abstract service at <http://adsabs.harvard.edu/abstract_service.html>. You can for example enter the name of an author in one field, the start and end year for your search, and obtain a list of all papers by that author, with links to the on-line journal contents. (You may need to be on campus or use the library proxy-service to actually get access to the journal.) As a hint that you've found the RIGHT paper, Putzig is the first author and the title begins "Subsurface structure of ..." .

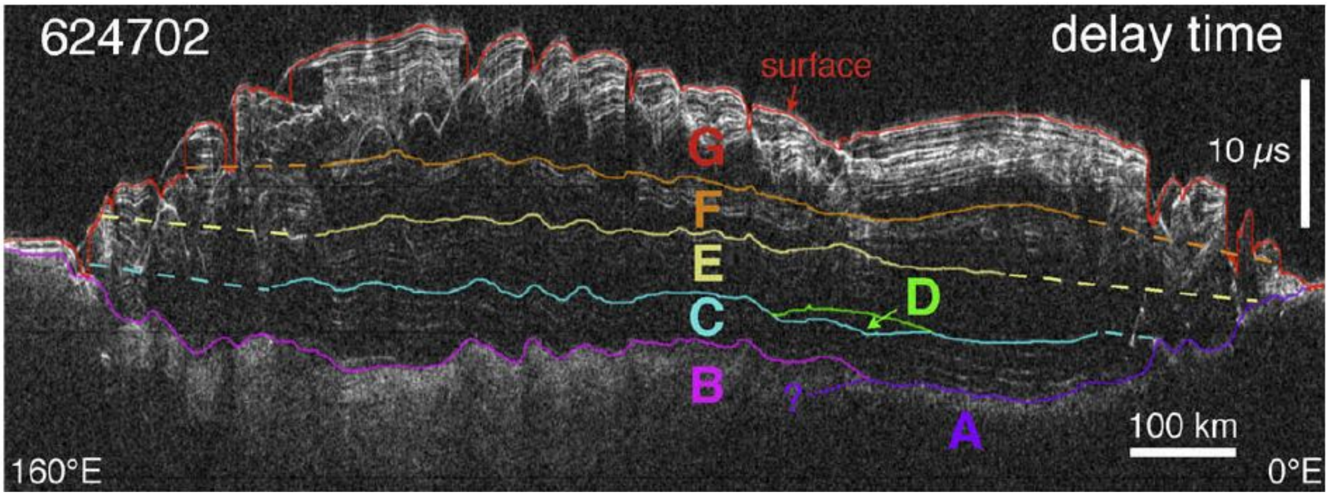
In their paper Putzig et al. give an equation which they use to correct from observed delay time (which is what the radar measures) to actual ice depth. In that same part of the paper they also give the value of the "real permittivity" (ε) for ice which goes into that equation. Report the equation and the permittivity value they use, and using their equation (perhaps inverted) calculate the actual two-way delay time which the radar system would observe between the top of the ice and the basal unit.

*Their equation (1) is* $\Delta d = c\, \Delta t/ (2\, \varepsilon^{1/2})$ *where I've dropped the subscript i and added the parentheses to be sure there isn't any confusion. They adopt a value of* $\varepsilon = 3.15$ *for ice. Using that formula and solving for* $\Delta t$ *gives* $\Delta t = 2\, \Delta d\ \varepsilon^{1/2}/c$ *which is just* $\varepsilon^{1/2} = (3.15)^{1/2} = 1.78$ *times greater than the formula we used in part B. So the observed delay time should actually be* $14 \times 1.78$ *microseconds = 25 microseconds.*

**D) (5 points)** In a part of the figure which goes with the above one I've shown, they actually show a version of the upper image, displayed with the vertical axis being delay time rather than distance. Compare your estimate with the delay time with what they actually see, which you can estimate from that plot.

*Inserted below is the upper part of their figure 3, which shows that the observed delay time is 23 microseconds -- equal the the above answer within measurement error.*

*Note that in a "delay time" image like this, which has NOT been corrected for the slowdown of the wave in ice, the top of the base unit (bottom of the ice cap) looks very rough, but that is an artifact caused by the extra delay due to the ice. The clue to this is each of the undulations there is a mirror image of those on the surface. In reality the base is relatively flat.*




**E) (5 points)** Earlier in the semester I mentioned the formula for speed of light in a substance was given by v = c / n, where c is the speed of light in a vacuum and n is the index of refraction. From their formula for v and the one above, solve for n in terms of the permittivity ε.

Finally, take a look at their full Figure 2, including the top panel which I've left off in the version I show above. The initial data plots from SHARAD typically just used delay time as the vertical axis, and in those plots, because of the extra delay introduced by the slower speed of light in ice, the top of the basal unit looked very rough -- in a way that was almost a mirror image of the surface ice topography. It is only after correction for the speed effects mentioned above that it became clear the top of the basal unit was actually much more uniform.

*Comparing our equation from part 1, rewritten in terms of* delay time *$\Delta t$ and one way distance $\Delta d$, and using true speed $V$, we have $\Delta t = 2\,\Delta d / V$ while their equation is equivalent to $\Delta t = 2\,\Delta d\ \varepsilon^{1/2}/c$ so*

*$V = 1/\varepsilon^{1/2}$. Since the formula for the index of refraction is $n = c / V$ we have:*

*$n = \varepsilon^{1/2} = 1.78$. The radar wave is essentially slowed down by the way the water molecules react to the passing electrical field. This radar index of refraction is larger than the 1.309 value at visual wavelengths, because at optical frequencies the polarized $H_2O$ molecule has less time to "respond" to the electric field of the EM wave. While ice doesn't have "magnetic" properties, if it did, we'd actually have another term in n which described how they responded to the magnetic field.*