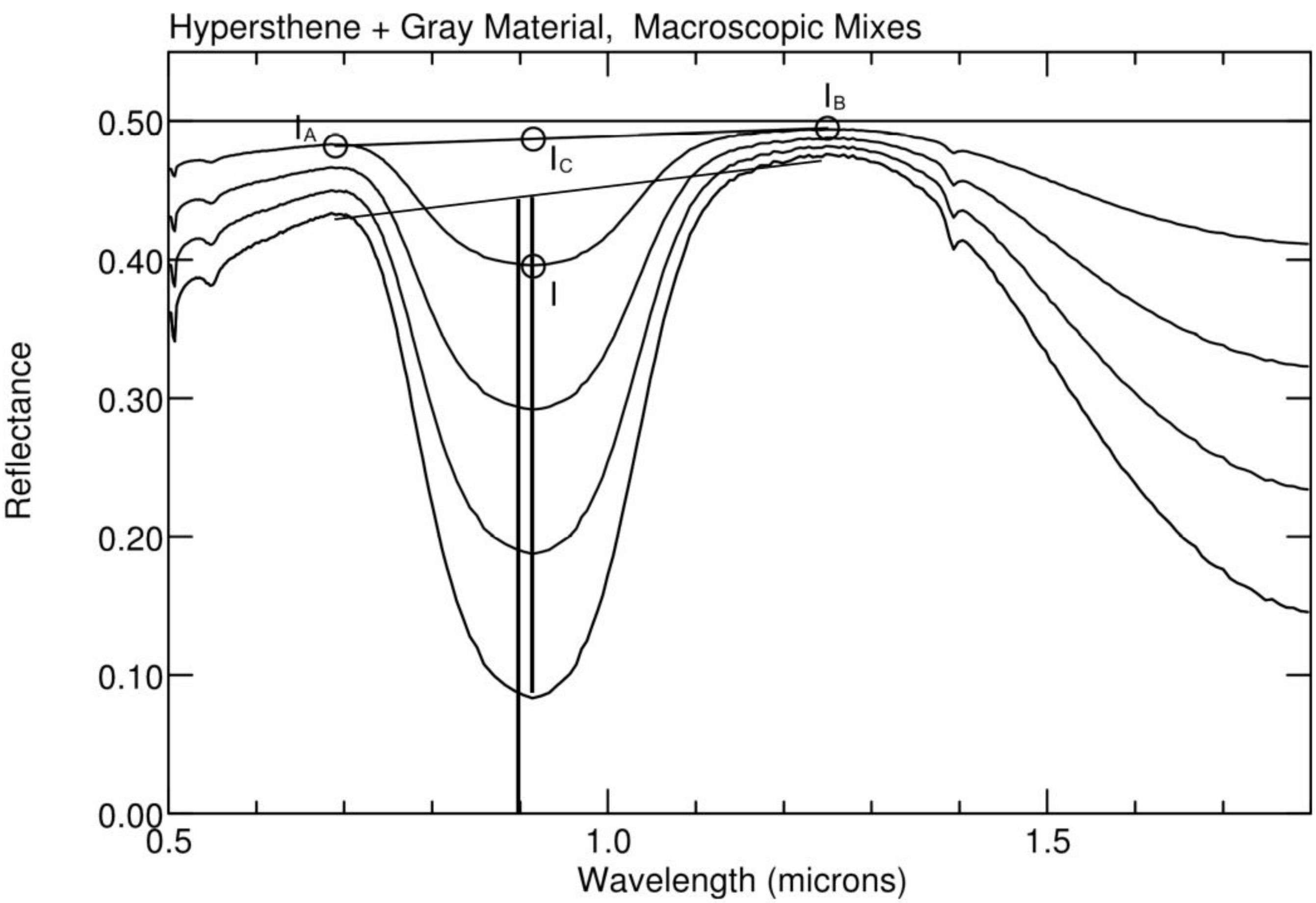
Homework #7 **SOLUTION**
Geology 4113 (Remote Sensing)
Assigned March 09, 2018
Due March 23, 2018

1) Band Depth Ratios and macroscopic mixtures (20 points)

Band Depth Ratios (band depths for short) are used to quantitatively measure the depth of absorption bands. They will range from 0 (no absorption) to 1 (complete absorption). When mixtures of materials are present they can be used to estimate abundances.

The graph below shows the spectra expected for a surface covered completely with 60-μm size grains of hypersthene (the bottom curve), a gray material with a reflectance of 0.50 (top curve) plus various macroscopic (i.e. large scale or checkerboard) mixes of the two.

Note: In the following plot on the "solution" I've drawn, for the bottom curve, the continuum slope and the two additional distances you need to measure. For each of the four spectra you should have drawn and measured those elements. It just gets to confusing if I show all four on the solution plot.



In the following you will find the Band Depth of the 0.915 μm band for the four curves showing absorption there. To do this you will find the continuum points I_A and I_B on each side, find the interpolated continuum value I_C at the band center (i.e. the I you would have if no band were present), and find the actual intensity at band center I. Then you compute the band depth from

BD = $(I_C\text{-}I)/I_C = 1\text{-}I/I_C$. The appropriate points to use are shown as open circles for the 2nd highest curve. Note the continuum points **are not** spaced symmetrically around the band for this problem, so you can't simply say that I_C is the average of those two continuum points as we will in lab #8.

Detailed instructions:

Part A. (10 points)
Find the band depth for all 4 curves graphically – by drawing lines and measuring distances on the plot. Note you don't need to convert those distances into reflectance, since you only care about ratios of distances. Show on the plot the distances you measure and record those numbers.

For example, for the top curve the distance from the deepest point on the band down to the y=0 line is 7.35 cm while the distance from the interpolated continuum point I_C down to the y=0 line is 9.02 cm, so the band depth is 1-(7.35/9.02) = 1 – 0.815 = 0.185. (The distances may be slightly different on the finally Xeroxed version of the homework.) You could also obtain this (with more work) by noting that I at the deepest part of the band is 0.395 while I_C is 0.484 so BD = 1 – (0.395/0.484) = 1-0.816 = 0.184, essentially the same without roundoff and measurement error. Repeat this for the lower 3 curves.

Measuring distances for Ic-I and Ic for the four bands I get

	Ic-I	*Ic*	*Band Depth = Ic-I/Ic*
#1	*1.70 mm*	*9.05 mm*	*0.188*
#2	*3.35 mm*	*8.80 mm*	*0.381*
#3	*5.05 mm*	*8.60 mm*	*0.587*
#4	*6.80 mm*	*8.35 mm*	*0.814*

Part B. (5 points)
When we are working with digital data, once we pick the bands which correspond to λ_A, λ_B, and λ, we can program our computers to automate this calculation. To be sure you understand the technique involved, for one curve (the lowest) we will perform the calculation manually.

For that lowest curve, find the actual value of the two continuum intensities I_A and I_B and also I at the deepest part of the band. Also find the corresponding wavelengths λ_A, λ_B, and λ. You should try to make these measurements accurate to a few percent by using a ruler and interpolating using the X and Y scales and using a calculator. For example in Y every cm corresponds to 0.054 units in I, at least before xeroxing. Since that may have changed in your copy, determine the scale yourself.

Next, compute the interpolated continuum intensity I_C at the deepest part of the line from the following formula, which is the equation of a line passing through points (λ_A, I_A) and (λ_B, I_B).

$$I_C = I_A + \frac{I_B - I_A}{\lambda_B - \lambda_A}(\lambda - \lambda_A)$$

Finally, using I and I_C, find the band depth. Does the result you obtained here agree with the value you obtained graphically in part A?

I measure I_A= 0.433 at λ_A = 0.69 μm, I_B = 0.474 at λ_B = 1.25 μm, and I = 0.089 at λ = 0.91 μm.

Using the above formula gives

$$
\begin{aligned}
I_C &= 0.433 + [(0.474 - 0.433)/(1.25 - 0.69)] \times (0.91 - 0.69) \\
&= 0.433 + 0.0732 \times 0.22 \\
&= 0.433 + 0.016 \\
&= 0.449
\end{aligned}
$$

This is the same as you get by measuring the I_C you obtained graphically in part A.
Now using these values BD = 1-I/I_C = 1 – 0.089/0.449 = 1 – 0.198 = 0.802

Within the accuracy of our measurements, this is the same as Part A's value of 0.814

If you were calculating band depths with already digitized data the methods of part B would be the ones to use.

Part C. (5 points)

If you have an areal mixture, with a fraction *f* of the surface covered with hypersthene, and the remaining fraction (1-*f*) covered with a gray material, then the band depth you observe will be equal to *f* times the band depth of pure hypersthene (the lowest curve).

$$\mathrm{BD}_{\mathrm{Observed}} = \mathrm{f} \times \mathrm{BD}_{\mathrm{Hyperthene}}$$

Use this to estimate the fractional hypersthene coverage for the three intermediate curves.

In an areal mix where f is the fraction of surface covered by a material causing the band, while the rest of the surface is gray material which produces no band, the final band depth will be

$BD = f \times BD_{full}$ *so*

$f = BD / BD_{full}$

We know BD_{full} is given by the bottom curve, #4, BD_{full} = 0.814 so

#1 f = 0.188 / 0.814 = 0.23
#2 f = 0.381 / 0.814 = 0.47
#3 f = 0.587 / 0.814 = 0.72

In fact when I created these spectra I used fractions of 0.25, 0.50, and 0.72. The small difference between those and the results is largely from the limited accuracy in measuring distances on the plot.