Homework #5 **Solution**
Geology 4113 (Remote Sensing)
Assigned Feb 23 2018
Due Mar 02, 2018

1) Spectral Mixing. (10 points) Suppose you observe that the reflectance of a surface is 0.58, and you know it is composed of a macroscopic mixture of snow with reflectance 0.9 and dirt with a reflectance of 0.1. Find the relative abundance (i.e. the fraction of the area) covered by the dirt and by the snow.

The fundamental equation for macroscopic mixing is
$I = R_1 \times f_1 + R_2 \times f_2$ *where* R_n *is the reflectance of the n'th component and* f_n *is its abundance. We also have the fact that* $f_1 + f_2 = 1$ *so* $f_2 = 1 - f_1$*, giving*
$I = R_1 \times f_1 + R_2 \times (1 - f_1)$ *or*
$I = (R_1 - R_2) \times f_1 + R_2$ *or*
$f_1 = (I - R_2) / (R_1 - R_2)$

In this case the above equation becomes $f_1 = (0.58 - 0.9) / (0.1 - 0.9) = -0.32 / (-0.8) = 0.40$ *and* $f_2 = 1 - f_1 = 1 - 0.40 = 0.60$ *so the surface is 40% dirt and 60% snow.*

2) Microscopic mixing

Part A) (5 points) If in the above problem the same proportions of snow and dirt were mixed microscopically would the reflectance be greater than or less than 0.58?

Part B) (5 points) To make a crude quantitative estimate of the reflectance of a 50%-50% microscopic mix surface (not exactly what you have) suppose photons are reflected once off the snow and then once off the dirt before being coming back towards the observer. Find the reflectance of that "double bounce" surface.

The darker component always dominates in microscopic mixing. Therefore the reflectace would ***be less*** *than the 0.58 indicated above if the 40% dirt / 60% snow mix was microscopic.*

To estimate the reflectance of a 50% mix, suppose the light bounces off each component once. The amount of incoming light is I_0 *then after the first bounce it is* $I_1 = I_0 \times R_1$*. Now use that as input to the second part. After the second bounce the amount of light will be* $I_2 = I_1 \times R_2 = I_0 \times R_1 \times R_2$*. Considered as a system, the total reflectance is* $I_2 / I_0 = (I_0 \times R_1 \times R_2)/I_0 = R_1 \times R_2$ *which in this case will be* $0.1 \times 0.9 = 0.09$*, actually darker than the dirt itself. In reality it probably won't be quite this dark as some of the light will hit just the snow, some will hit just the dirt, but most will hit both.*

A full calculation requires knowledge of exactly how well mixed the particles are and what their sizes are, then uses the equations which come from Hapke theory.

5.01 If $\lambda_{max} = 0.65\,\mu m$ then from the Wien Displacement Law

$$\lambda_{max} = \frac{2897\,\mu m\,K}{T_{red}}$$

$$T_{red} = \frac{2897\,\mu m\,K}{\lambda_{max}} = \frac{2897\,\mu m\,K}{0.65\,\mu m}$$

$$= 4457\,K.$$

5.02 Again using the Wien displacement law

$$\lambda_{max} = \frac{2897\,\mu m\,K}{7000\,K} = 0.414\,\mu m$$

(note - most real filaments are much cooler than this, so the light they give off is redder.)

5.03 $F_b = \sigma T^4$. but we need T in kelvin

$$21°C = (21° + 273°)\,K$$
$$= 294°K.$$

$$F_b = (5.67 \times 10^{-12}\,W\,cm^{-2}\,K^{-4}) \times (294\,K)^4$$
$$= 4.24 \times 10^{-2}\,W\,cm^{-2} \quad \text{or with } \times (10^2\,cm/m)^2$$
$$= 4.24 \times 10^2\,W\,m^{-2}$$

5.04 For rough dolomite from Table 5-1 $\epsilon = 0.958$

The $T_{kin.} = 15°C$ corresponds to $(15 + 273)\,K = 288\,K$

$$F = \epsilon\sigma T^4 = 0.958 \times 5.67 \times 10^{-12}\,W\,cm^{-2}\,K^{-4} \times (288\,K)^4$$
$$= 3.74 \times 10^{-2}\,W\,cm^{-2}$$
$$= 3.74 \times 10^2\,W\,m^{-2}$$

5.06 $P = \sqrt{K \rho C}$

$= \sqrt{0.005\ \text{cal cm}^{-1} s^{-1}\ {}^\circ C^{-1}\ 2.7\text{g cm}^{-3}\ 0.19 \text{cal g}^{-1}\ {}^\circ C^{-1}}$

$= \sqrt{0.0026\ \text{cal}^2\ \text{cm}^{-4}\ {}^\circ C^{-2}\ s^{-1}}$

$= 0.0506\ \text{cal cm}^{-2}\ {}^\circ C^{-1}\ s^{-1/2}$

To convert to SI units multiply by

$4.1868\ \text{J/cal} \times (10^2 \text{cm/m})^2$ to get

$= 2120\ \text{J m}^{-2}\ K^{-1}\ s^{-1/2}$

or $2120\ W\ m^{-2}\ K^{-1}\ s^{+1/2}$

where $W = J/s$ and a ΔT of $1\ {}^\circ C$ is the same as $1 K$.

5.07 $ATI = \frac{1-A}{\Delta T} = \frac{1-A}{20^\circ C - 10^\circ C} = \frac{1-0.5}{10^\circ C} = 0.05\ {}^\circ C^{-1}$