

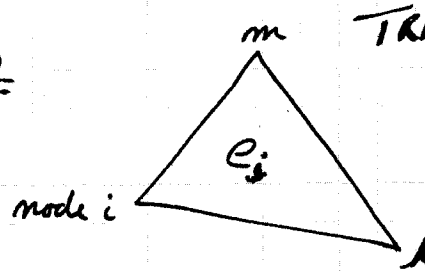
Date:

Class:

Assignment:

Integrals over element  $i$

$e_i$  has area  $A_i$



TRIANGLE ELEMENTS

nodes are ordered  
counterclockwise

$e_i$  has nodes at:

$(x_i, y_i) (x_k, y_k) (x_m, y_m)$  labeled counterclockwise

$$A_i = \frac{1}{2} |x_i(y_k - y_m) + x_k(y_m - y_i) + x_m(y_i - y_k)|$$

$$\langle \phi_i \rangle^{e_i} = \frac{A_i}{3}$$

$$\langle \phi_i \phi_i \rangle^{e_i} = \frac{A_i}{6}$$

$$\langle \phi_i \phi_k \rangle^{e_i} = \frac{A_i}{12}$$

$$\left\langle \left( \frac{\partial \phi_i}{\partial x} \right)^2 \right\rangle^{e_i} = \frac{1}{4A_i} (y_k - y_m)^2$$

$$\left\langle \left( \frac{\partial \phi_i}{\partial y} \right)^2 \right\rangle^{e_i} = \frac{1}{4A_i} (x_m - x_k)^2$$

$$\left\langle \frac{\partial \phi_i}{\partial x} \frac{\partial \phi_k}{\partial x} \right\rangle^{e_i} = \frac{1}{4A_i} (y_k - y_m)(y_m - y_i)$$

$$\left\langle \frac{\partial \phi_i}{\partial y} \frac{\partial \phi_k}{\partial y} \right\rangle^{e_i} = \frac{1}{4A_i} (x_m - x_k)(x_i - x_m)$$