A note on using the harmonic mean, instead of the arithmetic mean, for a thermal conductivity that varies between 2 nodes in a finite difference grid.

Consider 2 nodes (T0 and T1) with material between them that is ½ one conductivity (K1) and ½ is K2. The K1 material occupies half of the distance between T0 and T1, and K2 the other half. Heat flowing from T0 to T1 travels through K1 and then through K2. We assume steady state, so the flux of heat through the K1 region is the same as through the K2 region. The distance between T0 and T1 is delx.

Therefore q = -K1*(T1-Tm)/(delx/2) [egn 1] and q also = -K2*(Tm-T2)/(delx/2) [eqn 2], where Tm is the unknown temperature half way between T1 and T2, and delx/2 is the half length between T1 and T2.

Since both equal q, then (simplifying a little); K1*(T1-Tm) = K2*(Tm-T2). Or rewriting as an eqn for the unknown Tm = (K1*T1 + K2*T2)/(K1+K2).

Substitute this expression for the unknown Tm into [eqn 1] above gives a value for the effective conductivity between nodes T1 and T2: q = [(2*K1*K2)/(K1 + K2)] * (T1-T2)/delx, where the conductivity expressed in the square brackets is the harmonic mean of K1 and K2.