element-wise matrix

$$\left[\begin{array}{c|c|c} \langle\phi_0'\phi_0'\rangle^{e_0} & \langle\phi_1'\phi_0'\rangle^{e_0} & 0 \\ \hline \langle\phi_0'\phi_1'\rangle^{e_0} & \langle\phi_1'\phi_1'\rangle^{e_0}+\langle\phi_1'\phi_1'\rangle^{e_1} & \langle\phi_2'\phi_1'\rangle^{e_1} \\ \hline \langle\phi_0'\phi_2'\rangle^{e_0?} & \langle\phi_1'\phi_2'\rangle^{e_1} & \langle\phi_2'\phi_2'\rangle^{e_1} \end{array}\right]\begin{Bmatrix} a_0 \\ a_1 \\ a_2 \end{Bmatrix} = \begin{Bmatrix} \langle 3\phi_0\rangle^{e_0} + \phi_0 \frac{d\hat{T}}{dz}\Big|_0^3 \\ \langle 3\phi_1\rangle^{e_0} + \langle 3\phi_1\rangle^{e_1} + \phi_1 \frac{d\hat{T}}{dz}\Big|_0^3 \\ \langle 3\phi_2\rangle^{e_1} + \phi_2 \frac{d\hat{T}}{dz}\Big|_0^3 \end{Bmatrix}$$

$= 0$ in e_0?

terms like $\langle 3\phi_2\rangle^{e_1} \Rightarrow \int_1^3 3\left(-\tfrac{1}{2}+\frac{z}{2}\right)dz = \left(-\frac{3z}{2}+\frac{3z^2}{4}\right)\Big|_1^3$

$$= -\frac{9}{2}+\frac{27}{4}+\frac{3}{2}-\frac{3}{4}$$

finally $\langle 3\phi_2\rangle^{e_1} = 3$

and terms like $\phi_1 \frac{d\hat{T}}{dz}\Big|_0^3$ since $\phi_1 = 0$ @ $z = 0, 3$

then the term $= 0$

and $\phi_2 \frac{d\hat{T}}{dz}\Big|_0^3$ since $\phi_2 = 0$ @ $z = 0$ <u>and</u> B.C. $\frac{dT}{dz} = 0$ @ $z = 3$!

then this $= 0$

finally the $T(z=0) = 0$ B.C. requires $a_0 = 0$!

gives finally (I have filled in some of the $\langle\,\rangle^e$ integrals to help you!)

$$\begin{bmatrix} 1 & \langle\phi_1'\phi_0'\rangle^{e_0} & 0 \\ \langle\phi_0'\phi_1'\rangle^{e_0} & \langle\phi_1'\phi_1'\rangle^{e_0}+\langle\phi_1'\phi_1'\rangle^{e_1} & \langle\phi_2'\phi_1'\rangle^{e_1} \\ 0 & -1/2 & 1/2 \end{bmatrix}\begin{Bmatrix} a_0 \\ a_1 \\ a_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ \langle 3\phi_1\rangle^{e_0}+\langle 3\phi_1\rangle^{e_1} \\ 3 \end{Bmatrix}$$

fill in the missing integrals and solve.

$\boxed{\text{hint! } a_2 = 13.5^\circ C}$