

(4)

To be completely obvious, the $i=0$ eqn is

$$a_0 \langle \phi'_0, \phi'_0 \rangle + a_1 \langle \phi'_1, \phi'_0 \rangle + a_2 \langle \phi'_2, \phi'_0 \rangle - \langle 3, \phi_0 \rangle - \left. \frac{d\hat{T}}{dz} \phi_0 \right|_{z=0}^{z=3} = 0$$

and likewise the $i=1$ and 2 eqns

OR we can write as a linear algebra eqn set!

$$\begin{bmatrix} \langle \phi'_0, \phi'_0 \rangle & \langle \phi'_1, \phi'_0 \rangle & \langle \phi'_2, \phi'_0 \rangle \\ \langle \phi'_0, \phi'_1 \rangle & \langle \phi'_1, \phi'_1 \rangle & \langle \phi'_2, \phi'_1 \rangle \\ \langle \phi'_0, \phi'_2 \rangle & \langle \phi'_1, \phi'_2 \rangle & \langle \phi'_2, \phi'_2 \rangle \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} \langle 3, \phi_0 \rangle + \left. \phi_0 \frac{d\hat{T}}{dz} \right|_0^3 \\ \langle 3, \phi_1 \rangle + \left. \phi_1 \frac{d\hat{T}}{dz} \right|_0^3 \\ \langle 3, \phi_2 \rangle + \left. \phi_2 \frac{d\hat{T}}{dz} \right|_0^3 \end{bmatrix}$$

at this point all we need to do is calc the various integrals. Note many terms are zero since basis fns are zero over some of the domain so $\langle \phi'_0, \phi'_2 \rangle = 0$ since ϕ_0 and ϕ_2 don't overlap.

from page 1 $\phi'_0 = -1$, $\phi'_2 = \frac{1}{2}$ and $\phi'_1 = 1$ in e_0
 $= -\frac{1}{2}$ in e_1

so $\langle \phi'_0, \phi'_0 \rangle = \langle -1, -1 \rangle$ over $e_0 = 1$

$\langle \phi'_1, \phi'_1 \rangle$ requires for thought, it has values in both e_0 and e_1 so we could write it as

$$\langle \phi'_1, \phi'_1 \rangle = \langle \phi'_1, \phi'_1 \rangle^{e_0} + \langle \phi'_1, \phi'_1 \rangle^{e_1}$$

example
 one of the terms

$$\langle \phi'_1, \phi'_1 \rangle^{e_1} = \int_1^3 \left(-\frac{1}{2}\right) \cdot \left(-\frac{1}{2}\right) dz = \left. \frac{z}{4} \right|_{z=1}^{z=3} = \frac{1}{2}$$

it is useful to write the matrix eqn in terms of the elements

since we have to integrate terms that span 2 elements as the sum of 2 integrals