therefore rewrite as:

$$\left\langle \hat{T}'', \phi_i \right\rangle + \left\langle 3, \phi_i \right\rangle = 0$$

and "integrate by parts" on $\left\langle \hat{T}'', \phi_i \right\rangle$

aside int by parts

$$\int_x^y a\,db = ab\Big|_x^y - \int_x^y b\,da$$

for 2 fns a and b

let $\hat{T}'' = db$

$\phi_i = a$

then $$\int_0^3 \phi_i \hat{T}''\,dz = \hat{T}'\phi_i\Big|_0^3 - \int_0^3 \hat{T}'\phi_i'\,dz$$

or $$\left\langle \hat{T}'', \phi_i \right\rangle = -\left\langle \hat{T}', \phi_i' \right\rangle + \hat{T}'\phi_i\Big|_0^3$$

3 eqns in $i=0,1,2$

now we can sub in a more explicit form for $\hat{T}(z) = a_0\phi_0 + a_1\phi_1 + a_2\phi_2$

$$-\left\langle \hat{T}', \phi_i' \right\rangle \Rightarrow \left\langle a_0\phi_0', \phi_i' \right\rangle + \left\langle a_1\phi_1', \phi_i' \right\rangle + \left\langle a_2\phi_2', \phi_i' \right\rangle \quad i=0,1,2$$

note that the expansion gives a sum of 3 terms, and 3 separate eqns in $i=0,1,2$

we could write it as: $$\sum_{j=0}^{2} \left\langle a_j\phi_j', \phi_i' \right\rangle$$

NOTE j in the sum is the sum index, i is the equation index. They are different.

and since the a_j are not fns of z

$$\sum_{j=0}^{2} a_j \left\langle \phi_j', \phi_i' \right\rangle$$

<u>to recap</u> we have turned $\left\langle \hat{T}'' + 3, \phi_i \right\rangle = 0 \quad i=0,1,2$

into (multiply by -1) $$\sum_{j=0}^{2} a_j \left\langle \phi_j', \phi_i' \right\rangle - \left\langle 3, \phi_i \right\rangle - \frac{d\hat{T}}{dz}\phi_i\Bigg|_{z=0}^{z=3} = 0$$

we could write this out in detail, but it goes to 0 in this problem, so is not worth it.