A short note on simple error propagation

When you are dealing with data, you often have an idea of the size of the error. In the simplest case where all you know is an idea of the bounds on the error, then we express that as a data value plus or minus some error value (X +/- *e*), for example 8.5+/-0.5. If you then have to process the data using mathematical operations there are some simple rules of thumb to propagate the error along with the data.

Adding (or subtracting) two sets of data (say A and B with errors e_A and e_B), if the data are unrelated and you assume they are approximately guassian in their errors, their sum and error is approximately:

$$X \pm e = A + B \pm \sqrt{(e_A)^2 + (e_B)^2}$$

Two useful notes: if either error is more than twice the other then the error is just the larger of the 2 original errors. The other note is that when you subtract, you tend to magnify the error, since the data is reduced, but the error is not.

Multiplying two data sets, again unrelated and guassian:

$$X \pm e = A * B \pm \sqrt{\left(\frac{e_A}{A}\right)^2 + \left(\frac{e_B}{B}\right)^2}$$

Note if one of the errors is larger as a percent of the data, then the resulting error of the product is that same percentage of the product.

Or

Error after addition approximately the magnitude of the largest error
Error after multiplication approximately the largest percentage error

For rough work that is all that is needed, however sometimes the errors are not guassian, they may be skewed. Or the data is correlated and A has some dependence on B. Or the data processing is too complex to be approximated by simple math operations. For more difficult problems it is common to look to **Monte Carlo** methods to give a better estimate of errors.