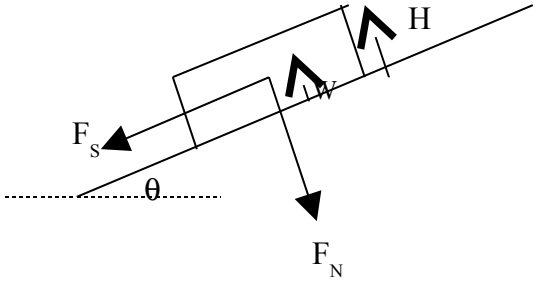
Geol 2150, Spring 09, Humphrey and Riebe
An Assessment of Hillslope Stability Using the Factor of Safety, plus:
A Sensitivity Analysis of the Balance between Driving and Resisting Forces.

Learning Goals:

- Predict hillslope stability using the Factor of Safety (FS)
- Assess which variables involved in calculating FS are mostly likely to change
- Construct an Excel spreadsheet to calculate FS
- Assess the sensitivity of FS to changes in hillslope parameters

Part I. Assessing Slope Stability Using Factor of Safety

Engineering geologists often use the relationship between driving shear stress (the component of stress that operates in the down-slope direction, σ_s) and potential resistive shear strength (the properties that resist shear stress, i.e., cohesion + potential frictional stress (which depends on the normal stress σ_N)) to carry out a slope stability analysis. As we discussed in class the ratio of shear strength to shear stress is called the factor of safety (FS). We can consider this ratio for the simplified case of a planar failure on an infinite slope as pictured below. (The infinite slope assumption makes sure we are not worrying about some buttress effect from a nearby dissimilar part of the slope). When this ratio is greater than 1, resistive shear strength is greater than driving shear stress and the slope is considered stable. When this ratio is close to 1, shear strength is nearly equal to shear stress and the slope is close to failure, if FS is less than 1 the slope should have already failed.



F_S = shear stress
F_N = normal stress
θ = slope angle
W = height of water table
H = thickness of soil

In this exercise, you will consider the different variables that go into calculating the factor of safety. You will then assess which variables are likely to change over the relatively short time scale of weeks to years and which variables are intrinsic to the slope or the material making up the slope. The primary goal of this exercise will then be to carry out a **Sensitivity Analysis**. This analysis will allow you to determine which variables most affect the factor of safety (and thus slope stability) when they change.

1. Write out the equation for the factor of safety and define all of the variables.

2. Tom Buchanan, the President of UW, has phoned the Geology Department because he is worried about his house. It is located on a steep slope in Richards Park. There is more development going on above his property and he is worried that funneling of groundwater from above might make his

property prone to a slump type failure. This class is going to conduct a slope stability analysis to see if he needs to get a geotechnical company involved.
After a site visit, you determine that you can get an approximate answer to Tom's question by considering the property behind Tom's house to be a 2.0 m-thick mass of regolith sitting on top of a bedrock surface with a slope of 19 degrees. The home is located at the base of this slope, on slightly lower angle ground. The foundation is tied to bedrock, but the worry is that if the slope starts to move, it will cause damage to the foundation walls and house. During the visit, you determine that the regolith is unsaturated. You also estimate the following additional parameters for this site:

regolith cohesion = 1100 N/m^2
regolith angle of internal friction = 21^o
density of regolith = 2000 kg/m^3
(density of water = 1000 kg/m3)

a. Calculate current values for resistive shear strength and driving shear stress on this slope. Show your work.

b. What is the value for the Factor of Safety? Would you consider the slope currently stable?

c. To see if Tom should be worried, let us check a worse case scenario. Let us assume that construction at the top of the slope will direct so much water (during a storm) onto the property that it becomes totally saturated to the surface. What is the stability of the slope in this case? Should Tom get expert help?

3. Considering Tom's situation, which **two** variables in the FS are most likely to change over time scales of interest to the Prez? Under what circumstances, and to what degree, would you expect these variables to change?

4. Which of the variables listed in question 2 are intrinsic to the materials found at the site, and which can vary at the site, and why?

5. Bonus question for the brave: Explain why all the depths, for our calculation of stresses, are normal to the slope, not to the horizontal? (hint, it has to do with force per unit area)

Part II. A Sensitivity Analysis
1. Our discussion in the first part indicates that changing some variables will change the FS. In many problems with several quantities that may change in value, it is very useful to know which variables cause large changes and which don't change the final outcome by very much. A sensitivity analysis can be performed to identify how each variable will change the result. In a sensitivity analysis, typically each variable in the problem is changed by a small percentage, and then the percentage change in the final result is observed. In preparation for our sensitivity analysis, you will develop an Excel spreadsheet to automatically calculate the factor of safety, and make repetitive calculations easy.

Open factor_of_saftety.xls and note that the parameters listed in Part I, question 2 have already been entered for you in column B. The variables are highlighted in yellow and constants are shown below. Click on the cell to the right of "shear stress" (B16) and note that an expression that will calculate the shear stress has already been entered for you. Note, you write equations in Excel by referring to the cell location of the variables, and entering the usual math operators (+, -, *, /). You can also use trig functions like SIN. Use lots of parentheses to ensure the correct order of operations. Using the provided equation as a guide or hint, write an equation for the cells to the right of "shear strength" and "factor of safety." Note that all angles will need to be converted to radians by multiplying by PI and dividing by 180. To do this in Excel you'll need to multiply by the quantity PI()/180. Also note, that your FS in cell B18 should equal your solution to question 2b in Part I.

Now we will conduct a sensitivity analysis. To accomplish this, you will change one hillslope parameter at a time, noting the new value of FS. The key to a sensitivity analysis is to change one parameter at a time. For this reason, **it is crucial to return the spreadsheet to the initial values in between each test**. It can be useful to consider your answers in terms of the percent change in FS that occurs. A good approach is to copy column B to column D, this will give you another column to do your calculations with changed parameters. Another good idea is to write an equation for cell C20 which subtracts D18 from B18 and multiply by 100, which will give you the percentage change in FS caused by your change in parameters.

2. Let's first consider the two variables that are most likely to change over short time scales, namely cohesion and water depth.

i. Cohesion:
By what percentage does the factor of safety change if the cohesion value is reduced by 10%? Does the slope become more or less stable? What is the percentage change in the FS?
Does the FS change by a larger or smaller percentage than the variable? Explain, using the mathematical relationships between variables in your explanation.

ii. Water Depth:
Set the water depth to 1m. By what percentage does the factor of safety change if the water depth increases by 10%? Does the slope become more or less stable, and by how much?
Does the FS change by a larger or small percentage than the cariable? Explain, using the mathematical relationships between variables in your explanation.

3. Now we will consider the more intrinsic variables, i.e. variables that are specific to a material or a site, but that could change from location to location. These include regolith cohesion, the angle of internal friction, density and slope.

i. Regolith Density:
By what percentage does the factor of safety change if the regolith density is reduced by 10%?
Does the slope become more or less stable? What is the percentage change in the FS?

Does the FS change by a larger or smaller percentage than the variable? Explain, using the mathematical relationships between variables in your explanation.

ii. Angle of Internal Friction

By what percentage does the factor of safety change if the internal angle of friction is reduced by 10%? Does the slope become more or less stable? What is the percentage change in the FS?
Does the FS change by a larger or smaller percentage than the variable? Explain, using the mathematical relationships between variables in your explanation.

.**iii. Slope:**

By what percentage does the factor of safety change if the slope is reduced by 10%? Does the slope become more or less stable? What is the percentage change in the FS?
Does the FS change by a larger or smaller percentage than the variable? Explain, using the mathematical relationships between variables in your explanation.

.**iv. Gravity:**

And just for completeness so we can compare failures on Earth with those on another planet, for example Mars, by what percentage does the factor of safety change if gravity is reduced by 10%? Does the slope become more or less stable? What is the percentage change in the FS?
Does the FS change by a larger or smaller percentage than the variable? Explain, using the mathematical relationships between variables in your explanation.

4. Provide a summary of the results of your sensitivity analysis. Your can do this as a table, with a column for the name of the variable, and a column for the % positive or negative change in the FS. Provide a written summary in which you use the results of your analysis to conclude which variables are most important in determining slope stability. Also address which factors included here are the least important.

5. Admittedly, the expression we have used for this exercise involves a necessary simplification of the factors that determine hillslope stability. What important assumptions underlie the FS equation you have used in this exercise? How do these assumptions affect the applicability of your analysis?

6. Despite the assumptions we have made, what can you say about *how* and *why* each of the human activities listed below will likely affect hillslope stability? You will need to extend your thinking qualitatively in one case.

a. Slope dewatering
b. Vegetation removal
c. Soil compaction during construction
d. Groundwater concentration from roof and pavement drainage

7. A sensitivity analysis is a good way of understanding the response of a complex system. You should be aware however, that an analysis of a non-linear system is much more difficult since the sensitivity depends on the size of the inputs. Our slope stability analysis is actually mildly non-linear because of the sine and cosine terms. To see this, compare the percentage change in the FS due to a 10% change in the slope, on a slope of 5 degrees and on a slope of 50 degrees.