STATISTICAL CHARACTERIZATION OF GRAIN-SIZE DISTRIBUTIONS IN SANDY FLUVIAL SYSTEMS

ELIZABETH A. HAJEK,1 SNEHALATA V. HUZURBAZAR,2 DAVID MOHRIG,3 RANIE M. LYNDS,1 AND PAUL L. HELLER1

1University of Wyoming, Department of Geology and Geophysics, 1000 East University Avenue, Laramie, Wyoming 82071, U.S.A.
2University of Wyoming, Department of Statistics, 1000 East University Avenue, Laramie, Wyoming 82071, U.S.A.
3Jackson School of Geosciences, Department of Geological Sciences, 1 University Station C1100, Austin, Texas 78712-0254, U.S.A.
e-mail: haje0009@umn.edu

ABSTRACT: Measured particle-size distributions are commonly reduced to one characteristic value (e.g., median grain diameter) that is used in sediment transport modeling and other analyses. These values are often interpolated from empirical distributions or from fitted distributions, usually assuming that observed grain-size populations are adequately represented by Gaussian or Normal distributions. In order to investigate the implications of this approach, we (1) statistically characterize grain-size distributions in samples of bed-material load, suspended load, and slackwater deposits from the sand-bedded Calamus, North Loup, and Niobrara rivers (Nebraska, USA), and (2) explore the potential impact of misfitting distributions on estimating percentile grain diameters. Although log-normal distributions are commonly used to characterize complete grain-size distributions in sedimentary systems, in this study, samples of transported sediment are best modeled with log-hyperbolic distributions, and slackwater deposits are best fitted by mixtures of distributions. Despite large overlaps in the grain sizes of bed-material-load and suspended-load samples, estimated parameters of fitted log-hyperbolic distributions show consistent differences between these samples across all rivers. Samples of bed-material load have higher modes and positive (coarse-grained) asymmetry, whereas suspended-load samples have lower modes and weaker asymmetry. Because it is has a general form, the log-hyperbolic distribution should adequately characterize unimodal grain-size samples because its parameters can yield both normal-shaped distributions as well as asymmetric distributions. In all three rivers, slackwater deposits contain the entire range of grain sizes present in suspension as well as a significant component of very fine-grained (< 0.02 mm) material that is not present in suspended-sediment samples. This suggests some degree of fractionated deposition of suspended sediment in areas of near-zero flow velocities. Ultimately, fitting parametric grain-size distributions to grain-size data can be a useful way to find effective particle-size values for use in sediment transport modeling and other studies. However, particularly with asymmetric grain-size distributions, fitting log-normal distributions to data may result in errors of estimated percentile grain sizes, which should be considered in studies relying on characteristic grain-diameter values.

INTRODUCTION

The distribution of grain sizes in sedimentary systems is largely a function of the distribution of available sediment and sediment-transport processes that sort and redistribute particles. In most sediment transport modeling full particle-size distributions are commonly reduced to one characteristic value (e.g., median or 95th percentile grain diameter; D50 and D95, respectively). Percentile grain diameters are often estimated from empirical distributions either by linearly interpolating between weight percents of measured bin sizes or by using probability-based methods which are often based on normal distributions. An alternative to interpolating between measured bin weight percents is to fit a parametric distribution to empirical data and, for example, estimate D50 directly from the fitted distribution. The goals of this study are to (1) characterize grain-size distributions (GSDs) from three modern rivers in order to understand how GSDs vary within and between sandy river systems and determine which parametric distributions best describe empirical data, and (2) explore the potential impact of misfitting distributions on estimating percentile grain diameters.

Understanding how GSDs relate to sediment transport and deposition in different settings is important both in modern systems and ancient deposits. Advances in analytical particle-size measurement have increased descriptive capabilities, while improved computing power has expanded practically available statistical models for data (Fieller et al. 1992). Work focused specifically on statistically characterizing GSDs in various sedimentary systems has resulted in differences with respect to which distributions best describe samples from different environments (Ghosh and Mazumder 1981; Fieller et al. 1984; Hartmann and Christiansen 1992; Sutherland and Lee 1994). For example, some authors have proposed that fluvial grain-size distributions can be sufficiently approximated by a log-normal distribution (e.g., Kothyari 1995; Kranck et al. 1996a, 1996b; Purkait 2002). In contrast, other work has shown that other distributions, including log-skew-Laplace and log-hyperbolic distributions, fit some sediment samples better than log-normal distributions (e.g., Barndorff-Nielsen et al. 1982; Fieller et al. 1984; Flenley et al. 1987; Fieller et al. 1991; Fieller et al. 1992). While some have proposed explanations for why certain parametric distributions fit better than others (e.g., Ghosh and Mazumder...
In an effort to better understand how GSDs can vary within and between sandy river systems, we statistically characterize sediment samples from three modern rivers in central Nebraska, USA. Samples of bed-material load, suspended load, and slackwater deposit were collected from the Niobrara, North Loup, and Calamus rivers. Grain sizes between 0.001 and 30 mm for each sample were measured with a Horiba LA-300 laser particle-size analyzer and Retsch Technology digital image-processing particle-size analyzer, and statistical GSDs were modeled using the program ShefSize (Robson et al. 1997). Samples of bed-material load and suspended load show unimodal distributions best fitted by log-hyperbolic models, and estimated parameters from fitted log-hyperbolic distributions provide useful comparisons of sediment samples within and between rivers. These results differ from those of previous studies where fluvial sediments were found to be fitted best by log-normal distributions (e.g., Kothyari 1995; Kranck et al. 1996a, 1996b; Purkait 2002). Slackwater deposits have distributions with a relatively abundant fine-grained ($\leq 0.02$ mm) fraction and are best modeled by mixture distributions. These samples have finer-grained modes than suspended-sediment samples, suggesting that the relationship between suspended load and suspended-load deposits is more complex than suggested in previous studies (e.g., Ghosh et al. 1986).

To demonstrate the potential effect of mischaracterizing GSDs, simulated distributions were generated from log-hyperbolic distributions with a range of asymmetry parameters observed in the data presented in this study. Percentile grain diameters ($D_5$, $D_{10}$, $D_{50}$, $D_{90}$, $D_{95}$) are estimated from log-normal distributions fitted to the simulated data. For log-hyperbolic distributions with moderate negative and positive asymmetry values, the $D_{50}$ estimated from fitted log-normal distributions actually represents the $D_{41}$ and $D_{59}$, respectively, of the simulated data. Misfits also occur in one tail of asymmetric distributions where log-normal estimates of $D_{10}$ and $D_{90}$ actually represent the $D_2$ and $D_{97}$, respectively, for positively and negatively asymmetrical simulated data. Whether these errors are significant depends the objectives of a particular study, but the potential for misestimation of percentile grain diameters should be an important consideration for sediment transport modeling or other studies relying on characteristic grain diameters.

**STUDY AREAS AND METHODS**

Sediment samples were collected from the Niobrara, North Loup, and Calamus rivers in east-central Nebraska (Fig. 1) during the summer of 2004. These rivers are similar in overall setting and scale but provide some differences to be compared. For example, the rivers have similar overall...
sediment loads (dominantly sand-sized), but two have much sandier source areas than the other. Additionally, these rivers span a range of mean annual discharges from ~8 to almost 50 m$^3$/s, and two of the rivers are braided and one meandering.

The Niobrara is a braided river (Fig. 1A) that originates in east-central Wyoming, flows eastward across northern Nebraska, and terminates at the Missouri River at the study area near Niobrara, Nebraska (Fig. 1). Sediment sources include Miocene and Pliocene continental deposits in western Nebraska and the Cretaceous Pierre Shale and Niobrara Chalk in eastern Nebraska (Watkins and Diffendal 1997). The Niobrara River maintains year-round base flow (Bristow et al. 1999) because it derives most of its discharge from groundwater (Bleed 1990). The mean annual discharge of the Niobrara River near Verdel, Nebraska (20 km upstream of the study locality) from 1959 through 2007 was 49 m$^3$/s (http://waterdata.usgs.gov/ne/nwis/rt).

The North Loup River has a braided planform (Fig. 1B) and is sourced in the Sand Hills of central Nebraska, making it a particularly sandy braided river. The river was studied near the town of Taylor, Nebraska (Fig. 1). The mean annual discharge at the study location from 1938 through 2007 was 14 m$^3$/s (http://waterdata.usgs.gov/ne/nwis/rt).

The Calamus River, like the North Loup River, is sourced in and entirely confined to the Sand Hills of central Nebraska. On the studied reach, upstream of the Calamus Reservoir near the town of Burwell, Nebraska, the Calamus River is primarily a sinuous single-thread sand-bedded channel with infrequent mid-channel bars (Fig. 1C). The Calamus River has been interpreted as containing braided reaches (Bridge et al. 1998) but is single thread in the study locality. The sinuosity of the Calamus River in the study area is 1.8. The mean annual discharge on the Calamus River recorded at the Burwell gauging station from 1941 to 1994 was 8.4 m$^3$/s (http://waterdata.usgs.gov/ne/nwis/rt), recording highest discharges during the spring months.

**Sediment Sampling**

Sediment sampling for this study was designed to capture the distribution of grain sizes being transported in the mobile bed and in suspension of each river, and also of sediment accumulating in areas of near-zero flow velocity (slack water) within active channels. After collection, all samples were allowed to settle in the sample container, the water was decanted, and the sediment was placed into plastic bags for storage.

Samples of bed-material load were collected from the mobile channel bed as grab samples from active ripples (Fig. 2A), dunes, planar beds, and antidunes. Grab samples of mobile bed material were skimmed directly from the bed surface into a plastic sample container. These samples include sediment transported solely along the bed as well as suspended material that was temporarily resting on the bed and/or trapped interstitially amongst bedload particles.

Suspended-load samples were collected with a wading-type suspended-sediment sampler, US DH-48. The sampler is 33 cm long, consists of streamlined aluminum casting, and holds a plastic 0.47-liter sample container. Suspended sediment was collected from several locations in the high-velocity cores of the modern rivers: at roughly 0.2 times flow depth beneath the water surface and at 10 cm downstream from dune or bar crests at the submerged elevation of the crestline. These particular locations were selected in an effort to normalize for spatial variations in suspended-sediment size and concentration within the rivers due to flow depth and proximity to bedforms. In some cases the sampler did not catch enough sediment for analysis, so the processes was repeated a maximum of three times in the same location.

Slackwater deposits were sampled from areas within the active river channel with locally zero or near-zero flow velocities where fine-grained suspended sediment was accumulating (Fig. 2B, C). Samples were collected in plastic sample containers by skimming the upper ~1 cm.
of sediment from the river bottom in areas with suspended sediment deposits with thicknesses in excess of ~ 10 cm.

In addition to the goal of characterizing the GSDs associated with bed-load and suspended-load transport modes in the rivers, this sampling strategy was chosen so that modern data were collected in such a way that analogous material could be sampled from ancient outcrop or subsurface deposits for future comparisons. For example, samples of ancient bed-maternal load could be collected from preserved ripple or dune cross beds, and intrachannel slackwater deposits (Lynds and Hajek 2006) could be sampled and potentially used as a proxy for ancient suspended sediment loads.

Sample Preparation

All samples were dried by air or in a drying oven. Nearly half the samples contained significant amounts of organic material. The organic matter was removed by placing the sample in a beaker with twice the volume of water as sediment, and a 2:1 ratio of water to 30% hydrogen peroxide. The beaker with sediment, water, and hydrogen peroxide was placed under a fume hood on a hot plate at 40 °C and allowed to boil until dry. In a few cases this process was repeated twice to remove all the organic material.

All bed-material-load samples and some slackwater-deposit samples were divided into two populations sieved at 90 μm in preparation for subsequent grain-size measurement. Samples were sieved for approximately 15 minutes with a Ro-tap. The coarse-grained (> 0.09 mm) and fine-grained (< 0.09 mm) fractions were individually weighed and placed in plastic bags.

Grain-Size Assessment

In order to fully measure the wide range of grain sizes present in the samples, a Horiba LA-300 laser particle-size analyzer (LPSA) was used to measure the fine-grained (< 0.09 mm) fraction of each sediment sample, and the coarse-grained material (> 0.09 mm) was analyzed using a Retsch Technology digital image-processing particle-size analyzer (CAMSIZER). The LPSA uses a diode laser to measure grain sizes from 0.001 to 0.1 mm in diameter, and the CAMSIZER uses digital photographic images to measure grain sizes ranging from 0.05 to 30 mm in diameter, providing excellent characterization of the fine and coarse tails of the distribution, respectively. Both machines measure distributions as mass fraction of the sample, so the full grain-size distributions reported herein are the weighted average of the fine- and coarse-grained measurements.

STATISTICAL CHARACTERIZATION

Normal distributions are common in natural processes, and GSDs are often presumed to follow normal distributions in log scale (Middleton 1970). The log-normal distribution is a specific sub-case of the more general log-hyperbolic distribution. Barndorff-Nielsen (1977) formalized statistical modeling of GSDs by developing the hyperbolic family of distributions and using them to characterize data from eolian sediment deposits documented by Bagnold (1954). The log-hyperbolic distribution plots as a hyperbola on a log-log scale and so can reflect varying degrees of asymmetry, unlike the log-normal distribution, which plots as a parabola and is limited to being symmetrical. For this reason, the log-hyperbolic distribution can better represent asymmetry or skewness exhibited by many empirical GSDs.

The log-hyperbolic probability density function for x as the natural log of grain size is given by

$$g(x; \pi, \zeta, \delta, \mu) = \frac{\exp\{-\zeta[\sqrt{1 + \pi^2} \sqrt{1 + \left(\frac{x - \mu}{\delta}\right)^2} - \pi(\frac{x - \mu}{\delta})]\}}{2\delta\sqrt{1 + \pi^2}K_1(\zeta)}$$

for $x \in R$, where $K_1(\zeta)$ is the modified Bessel function of the third kind. The parameters are $\pi \in R$ indicating the asymmetry of the distribution, $\zeta > 0$ indicating the peakedness, and the location and scale parameter are given by $\mu \in R$ and $\delta > 0$, respectively. The case when this general distribution approaches the log-normal distribution occurs when $\pi = 0$, $\zeta \rightarrow \sigma^2$ as $\delta \rightarrow 0$ where $\sigma^2$ is the variance of the normal. Another special case, the log-skew-Laplace distribution, is defined when $\pi \neq 0$ and $\delta \rightarrow 0$, and this distribution plots as a pair of intersecting lines in log-log space.

Several studies have shown examples of log-hyperbolic or log-skew-Laplace GSDs in natural systems. Bagnold and Barndorff-Nielsen (1980) present examples of various samples, including river-bed sediments, that are best characterized by the log-hyperbolic distribution. Barndorff-Nielsen et al. (1982) used the log-hyperbolic distribution to examine variation in grain size in a sequence of sand samples along an eolian dune, as did Sutherland and Lee (1994) in their characterization of Hawaiian beach sands. The log-skew-Laplace distribution was found to be the best model for littoral sands from the Hebrides (Fieller et al. 1992) and Libyan eolian sediments (Flenley et al. 1987).

In all these examples, as well as in this and most other grain-size studies, data are grouped into classes of grain sizes. Furthermore, the exact number of grains in each class is not available, rather the weight percent of the sample within each class is recorded. Given the unknown number of grains in each class and in the total sample, in statistical analysis the likelihood function in the estimation procedure is replaced with the likelihood function (Barndorff-Nielsen 1977).

For this study, we use ShefSize (Robson et al. 1997), a statistical program developed to fit the log-hyperbolic, log-skew-Laplace, log-normal, and mixtures of two log-skew-Laplace distributions to each sample. When fitting distributions, we follow Fieller et al. (1992) and use a modified chi-squared-type statistic called $N_{cr}^i$ for comparing models:

$$N_{cr} = \frac{X^2}{\sum r_ip_i(\bar{p}/p_i)^2} \quad (2)$$

where $k$ is the number of grain-size classes, $m$ is the number of estimated parameters for a distribution (e.g., two for log normal and four for log hyperbolic), $i$ is the observed weight percent in the $i$th class, $p_i$ is the estimated probability (or estimated weight percent) of the $i$th class, and $\theta$ contains the parameters for the fitted distribution. Larger values of $N_{cr}$ indicate better model fit. Presently the log-hyperbolic, log-skew-Laplace, log-normal, and mixtures of two log-skew-Laplace are the only distributions available to fit in ShefSize, and they are sufficient for an exploratory analysis of the data.

RESULTS

Model Fitting

Across all three rivers, bed-load and suspended-load samples show single-peak distributions whereas slackwater deposits have a significant fine-grained tail (Fig. 3). In Table 1 we present information on the best-fitting distribution from the four available using ShefSize. In general, bed-material-load and suspended-sediment samples are best fitted using a log-hyperbolic distribution, with a portion of the samples being better fitted by log-skew-Laplace distributions and log-hyperbolic distributions, respectively. Slackwater-deposit samples tend to be multimodal and are better fitted using a mixture of two log-Laplace distributions.

Unimodal Distributions: Estimated Parameters

Estimated parameters from fitted log-hyperbolic distributions show consistent differences between grain-size distributions within and between
rivers. Estimated mode values for bed-material-load samples are, in
general, higher than those of suspended-sediment samples within each
river (Fig. 4). Likewise, the North Loup River has the highest average
bed-material-load modes as well as the highest average suspended-load
modes. Estimated asymmetry parameters (Fig. 5) also show differences
between bed-material-load and suspended-load deposits, where bedload
samples tend to be skewed toward coarser-grained sediment and
suspended-load samples tend to be symmetric or slightly skewed toward
finer-grained particles.

Mixture Distributions

All slackwater-deposit samples were better fitted by mixtures of two
log-skew-Laplace distributions than by any single distribution. This is
likely because of the significant fine-grained tail present in these samples.
Currently, there are no available routines for fitting mixtures of log-
hyperbolic models to weighted GSDs; however, once these are coded, it
will be worth exploring whether mixed log-hyperbolic distributions
provide a better fit than mixtures of log-skew Laplace distributions.

### Table 1.—Summary of best-fitting distributions for each sample type from each river. Log-hyperbolic distributions fit unimodal bed-material-load and suspended-load samples best, whereas slackwater-deposit samples are best fitted with a mixture of two distributions. Special-case log-hyperbolic column summarizes the number of samples best fitted with either log-normal distributions (“L. Norm.”) or log-skew-Laplace distributions (“L.S.L.”).

<table>
<thead>
<tr>
<th>Sample Type</th>
<th>Total Samples</th>
<th>Log Hyperbolic</th>
<th>Special Case Log Hyperbolic</th>
<th>Mixture Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Bed Load</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Calamus</td>
<td>20</td>
<td>12 (60%)</td>
<td>4 (20%) – L.S.L.</td>
<td>4 (20%)</td>
</tr>
<tr>
<td>Niobrara</td>
<td>33</td>
<td>27 (82%)</td>
<td>5 (15%) – L.S.L.</td>
<td>1 (3%)</td>
</tr>
<tr>
<td>North Loup</td>
<td>14</td>
<td>9 (64%)</td>
<td>5 (36%) – L.S.L.</td>
<td>0</td>
</tr>
<tr>
<td><strong>Suspended Sediment</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Calamus</td>
<td>12</td>
<td>8 (67%)</td>
<td>3 (25%) – L. Norm.</td>
<td>1 (8.3%)</td>
</tr>
<tr>
<td>Niobrara</td>
<td>18</td>
<td>17 (94%)</td>
<td>1 (6%) – L. Norm.</td>
<td>0</td>
</tr>
<tr>
<td>North Loup</td>
<td>9</td>
<td>7 (78%)</td>
<td>2 (22%) – L. Norm.</td>
<td>0</td>
</tr>
<tr>
<td><strong>Slackwater Deposits</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Calamus</td>
<td>14</td>
<td>0</td>
<td>1 (7%) – L.S.L.</td>
<td>13 (93%)</td>
</tr>
<tr>
<td>Niobrara</td>
<td>6</td>
<td>0</td>
<td>0</td>
<td>6 (100%)</td>
</tr>
<tr>
<td>North Loup</td>
<td>7</td>
<td>2 (29%)</td>
<td>0</td>
<td>5 (71%)</td>
</tr>
</tbody>
</table>
Historically, mixtures of distributions have been used to model grain-size distributions in natural systems. The reasons why sedimentologists use mixture distributions are that they provide tools for sedimentologists to easily fit distributions to sample data, compare goodness-of-fit for different distributions, and estimate distribution parameters.

Empirical relationships between, for example, bed material and suspended material or sorting and river slope (Kothyari 1995) based on presumed log-normality may be insufficient to characterize the variability of sediment distributions in natural systems. In particular, relationships of mode and variance used in models may be inconsistent for highly skewed or asymmetric samples, as well as samples showing mixture distributions. Having the full suite of parameters derived from fitting the log-hyperbolic distribution provides a good basis for statistical comparison and modeling (Hartmann and Flemming 2002).

**Log-Normal vs. Other Unimodal Distributions.**—The reasons why particular deposits are best characterized by log-hyperbolic rather than log-normal distributions remain unclear. For example, Purkait and Mazumder (2000) suggest that transport distance can transform originally log-hyperbolic samples into log-normal distributions—even on the scale of individual barforms. In this study, however, multiple samples from a variety of locations within reaches of three rivers consistently show log-hyperbolic distributions in both bed-material load and suspended load, regardless of position within channels or along bars.

In general, the log-hyperbolic distribution should provide a good fit to most samples, inasmuch as even log-normal distributions can be parameterized using log-hyperbolic distributions. In this study the result that some suspended-sediment samples are better fitted by log-normal distributions (Table 1) is probably due to the tendency for these samples to be symmetrical (Fig. 5), whereas the log-skew-Laplace function better captures the asymmetry exhibited by some of the bed-material-load samples.

**Mixture Distributions.**—Historically, mixtures of distributions have also been computationally difficult to fit (Fieller et al. 1991). Often, for the sake of simplicity, volumetrically less-significant grain-size fractions are ignored and only the dominant peak is modeled. For example, when analyzing suspended-sediment samples, Krang et al. (1996a, 1996b) focused on the main peak in the distribution and omitted the fine-grained “floc tail” for modeling sediment concentrations. They point out that ignoring this particular sediment fraction does not affect the mode of suspended-sediment samples, even in cases of repeated resuspension and deposition (Kranck et al. 1996b).

In this study, results show that mixture distributions better characterize slackwater deposits than any of the unimodal distributions. This is not
It is important to note that, unlike the distribution predicted by the Rouse equation, the entire proportion of fine-grained material carried in the channel. Although this material is not well represented in suspension, it can accumulate on the river bed, presumably along with colloids and contaminants transported with this grain-size fraction.

One possibility is that the slackwater deposits reflect sediment transported in the river under different flow conditions, a much higher proportion of fine-grained material was carried in the channel. This is unlikely, however, given that all three rivers show the same character and have different sediment sources and seasonal flooding histories.

Flocculation of fine-grained particles provides a potential mechanism for deviating from the unimodal log-hyperbolic GSD in a manner similar to that found in slackwater deposits. The observed slackwater deposits have distributions very similar to suspended-sediment samples of Kranck et al. (1996a, 1996b), whose sample distributions (measured as actual percentile grain diameters). Table 2 summarizes the discrepancy obtained from the fitted log-normal distributions and compared to the fitted log-hyperbolic GSD and that the tails do not match well.

Suspended Load and Slackwater Deposits.—The GSDs of slackwater-deposit samples in this study are striking in that they contain a significant proportion of fine-grained material (<0.02 mm), particularly with respect to what is being carried in suspension (Fig. 3). Somehow the finest grain sizes being transported in suspension are being concentrated in areas of near-zero flow velocity, suggesting some sort of fractionation, or preferential fine-grained deposition in these parts of the active channel. Although this material is not well represented in suspension, it can accumulate on the river bed, presumably along with colloids and contaminants transported with this grain-size fraction.

One possibility is that the slackwater deposits reflect sediment transported in the river under different flow conditions, a much higher proportion of fine-grained material was carried in the channel. This is unlikely, however, given that all three rivers show the same character and have different sediment sources and seasonal flooding histories.

Flocculation of fine-grained particles provides a potential mechanism for deviating from the unimodal log-hyperbolic GSD in a manner similar to that found in slackwater deposits. The observed slackwater deposits have distributions very similar to suspended-sediment samples of Kranck et al. (1996a, 1996b), whose sample distributions (measured as concentration rather than weight percent) show a main, relatively coarse peak that appears to be a unimodal distribution, and a relatively flat fine-grained (≤ medium silt) “floc tail” (Kranck et al. 1996a, 1996b). Flocculated material behaving as a single sand-size grain may contain unsorted particles ranging in size from clay to silt that are independently measurable upon disaggregation. Although we are uncertain the extent to which flocculation predominates in the studied rivers, this behavior is observed in fresh waters and occurs at a range of fine-grained sediment concentrations and flow velocities (e.g., Schieber et al. 2007).

One important motivation for understanding the relationship between suspended-sediment distribution and slackwater deposits is the possibility of reconstructing paleoflow conditions, including paleochannel slope or paleoflow velocity, from ancient deposits. Because of their work showing that, unlike the distribution predicted by the Rouse equation, the entire range of grain sizes in suspension are deposited on the bed, Ghosh and Mazumder (1981) and Ghosh et al. (1986) suggest that paleoflow velocity, in terms of bed shear velocity, could be calculated using the mode of suspended-sediment deposits and measures of local paleoflow depths. Samples from this study, however, show distinctly lower modes for suspended-sediment deposits (slackwater deposits) than material in transport as suspended load (Fig. 6). In the three rivers studied, differences in GSDs between suspended sediment and slackwater deposits are the result of spatial variation in sediment transport and depositional properties. These spatial variations, common in channels with irregular bottom topography, are not captured by the laboratory experiments of Ghosh et al. (1986). Future work will focus on modeling the differences between bedload, suspended load, and slackwater grain-size distributions in riverine deposits in an effort to establish statistically meaningful relationships between these populations.

**Potential Effects of Misfitting GSDs**

Commonly used sediment-transport equations often rely on single grain diameters (e.g., D50, D84, D95, etc.) calculated from empirical or fitted parametric GSDs. To explore the potential impact of misclassifying GSDs, we simulated data from log-hyperbolic distributions with varying degrees of asymmetry and fitted log-normal distributions to the simulated data. First, in Figure 7 we show an example of a Calamus River bed-load sample that is best fitted by a log-hyperbolic distribution. Note that the mode of the fitted log-normal distribution is distinctly offset from the fitted log-hyperbolic GSD and that the tails do not match well.

For data simulated from log-hyperbolic distributions, estimates of the mode and fitted parametric GSDs. To explore the potential impact of misclassifying GSDs, we simulated data from log-hyperbolic distributions with varying degrees of asymmetry and fitted log-normal distributions to the simulated data. First, in Figure 7 we show an example of a Calamus River bed-load sample that is best fitted by a log-hyperbolic distribution. Note that the mode of the fitted log-normal distribution is distinctly offset from the fitted log-hyperbolic GSD and that the tails do not match well.

For data simulated from log-hyperbolic distributions, estimates of the mode and fitted parametric GSDs. To explore the potential impact of misclassifying GSDs, we simulated data from log-hyperbolic distributions with varying degrees of asymmetry and fitted log-normal distributions to the simulated data. First, in Figure 7 we show an example of a Calamus River bed-load sample that is best fitted by a log-hyperbolic distribution. Note that the mode of the fitted log-normal distribution is distinctly offset from the fitted log-hyperbolic GSD and that the tails do not match well.

For data simulated from log-hyperbolic distributions, estimates of the mode and fitted parametric GSDs. To explore the potential impact of misclassifying GSDs, we simulated data from log-hyperbolic distributions with varying degrees of asymmetry and fitted log-normal distributions to the simulated data. First, in Figure 7 we show an example of a Calamus River bed-load sample that is best fitted by a log-hyperbolic distribution. Note that the mode of the fitted log-normal distribution is distinctly offset from the fitted log-hyperbolic GSD and that the tails do not match well.

For data simulated from log-hyperbolic distributions, estimates of the mode and fitted parametric GSDs. To explore the potential impact of misclassifying GSDs, we simulated data from log-hyperbolic distributions with varying degrees of asymmetry and fitted log-normal distributions to the simulated data. First, in Figure 7 we show an example of a Calamus River bed-load sample that is best fitted by a log-hyperbolic distribution. Note that the mode of the fitted log-normal distribution is distinctly offset from the fitted log-hyperbolic GSD and that the tails do not match well.

For data simulated from log-hyperbolic distributions, estimates of the mode and fitted parametric GSDs. To explore the potential impact of misclassifying GSDs, we simulated data from log-hyperbolic distributions with varying degrees of asymmetry and fitted log-normal distributions to the simulated data. First, in Figure 7 we show an example of a Calamus River bed-load sample that is best fitted by a log-hyperbolic distribution. Note that the mode of the fitted log-normal distribution is distinctly offset from the fitted log-hyperbolic GSD and that the tails do not match well.
This exercise demonstrates that if a sediment population has a log-hyperbolic GSD with some degree of asymmetry (e.g., $\pi \geq 0.5$ or $\pi \leq -0.5$, as do many of the bed-load and suspended-load samples in this study) the D50 calculated from a normal distribution would actually represent the D41 or D59 (depending on the direction of asymmetry) of the sample. Likewise, when the asymmetry parameter is negative (GSD is skewed to the fine tail) percentile grain diameters are overestimated in the coarse tail, with the inverse being true for distributions with positive asymmetry.

Whether using the D41 rather than the true D50 is a significant error depends on the precision required of a particular study and the scale of inquiry. There are cases where mischaracterizing GSDs could result in order-of-magnitude differences in results, including predicting changes to porosity and permeability from particle-size distributions (e.g., Beard and Weyl 1973). This should be decided on a case-by-case basis, but the potential error incurred by assuming normality should be taken into consideration.

**CONCLUSIONS**

Efficiently summarizing GSDs with parametric distributions can be useful for sediment transport modeling and other studies. We find that a log-hyperbolic distribution fits best for bed-material-load and suspended-load samples from the three sand-bedded rivers in this study. For other systems this may not always be the case, although because it is the most general distribution, the log-hyperbolic distribution should provide adequate estimates for a variety of unimodal grain-size samples, including those that are log-normally distributed. This is particularly important considering that the most commonly used distribution (log-normal) may poorly represent some aspects of data from samples, particularly those with strong asymmetry.

Slackwater deposits in this study are best characterized as a mixture, possibly of two log-skew-Laplace distributions, and show an increased proportion of fine-grained material relative to suspended sediment. These data also show that previous studies indicating that the suspended-sediment mode does not change with deposition are not universally true, especially in systems where bar-form topography induce large-scale spatial variations in flow velocity. Modeled relationships between suspended-load and suspended-deposit distributions must be established before paleoflow velocity or paleoslope can be calculated from ancient river deposits.

Modeling with simulated GSDs shows that estimates of median grain diameter from normal distributions fitted to populations with moderate negative or positive asymmetry actually represent the 41st or 59th percentile grain diameter, respectively, of a sample. Such errors in estimation of percentile grain diameter should be considered for studies in which GDSs are reduced to one characteristic grain diameter for sediment-transport modeling.

The availability of programs that can be used to fit statistical distributions to samples (including ShefSize) makes it easy for those working with sediment samples to be more quantitative about describing and comparing grain-size distributions. This, in turn, will improve our abilities to fully characterize the relationships between grain-size distributions and sediment transport processes.

**ACKNOWLEDGMENTS**

This research was partially supported by grants from the National Science Foundation (EAR-0345366) and the American Chemical Society Petroleum Research Fund as well as Geological Society of America, American Association of Petroleum Geologists, and International Association of Mathematical Geosciences foundation grants to R. Lynds. We thank Laura

---

**Table 2.** Comparison of percentile grain diameters estimated from fitted log-normal distributions to the true grain-diameter percentile of simulated data.

Five simulated log-hyperbolic GSDs were generated with varying degrees of asymmetry ($\pi = -1.0, -0.7, 0.0, 0.5,$ and $1.2$) and with the remaining three log-hyperbolic parameters held constant ($\mu = 4.5, \delta = 0.35,$ and $\zeta = 1.6$). The chosen parameters are within the observed range from bed-load and suspended-load samples in this study. Each row lists the true percentile of the grain diameter estimated as the D5, D10, D16, D50, D84, D90, and D95 from the fitted log-normal distribution for each simulated data set. For example, the median grain diameter (D50) estimated from a log-normal distribution fitted to the first simulation (a log-hyperbolic distribution with asymmetry $\pi = -1.0$) is actually the D41 of the simulated distribution.

<table>
<thead>
<tr>
<th>$\pi$</th>
<th>D5</th>
<th>D10</th>
<th>D16</th>
<th>D50</th>
<th>D84</th>
<th>D90</th>
<th>D95</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-1.0$</td>
<td>7</td>
<td>10</td>
<td>14</td>
<td>41</td>
<td>89</td>
<td>97</td>
<td>100</td>
</tr>
<tr>
<td>$-0.7$</td>
<td>7</td>
<td>10</td>
<td>14</td>
<td>42</td>
<td>88</td>
<td>95</td>
<td>99</td>
</tr>
<tr>
<td>0.0</td>
<td>5</td>
<td>9</td>
<td>14</td>
<td>50</td>
<td>86</td>
<td>91</td>
<td>95</td>
</tr>
<tr>
<td>0.5</td>
<td>2</td>
<td>6</td>
<td>12</td>
<td>57</td>
<td>86</td>
<td>90</td>
<td>94</td>
</tr>
<tr>
<td>1.2</td>
<td>0</td>
<td>2</td>
<td>10</td>
<td>59</td>
<td>86</td>
<td>90</td>
<td>93</td>
</tr>
</tbody>
</table>
Vietti for help in the field and are particularly appreciative of conscientious reviews by Whitney Autin, Gert Jan Weltje, and an anonymous reviewer.

REFERENCES


Received 19 October 2008; accepted 14 September 2009.