Mon. Oct. 30, 2017

- Reminder -- extra lecture Friday Nov. 3 at noon
- “Midterm” exam Wed. Nov. 8
- Reading: Ch. 5 (Atmospheres) from Wood

- Today:
  - Atmospheres Pt. 1

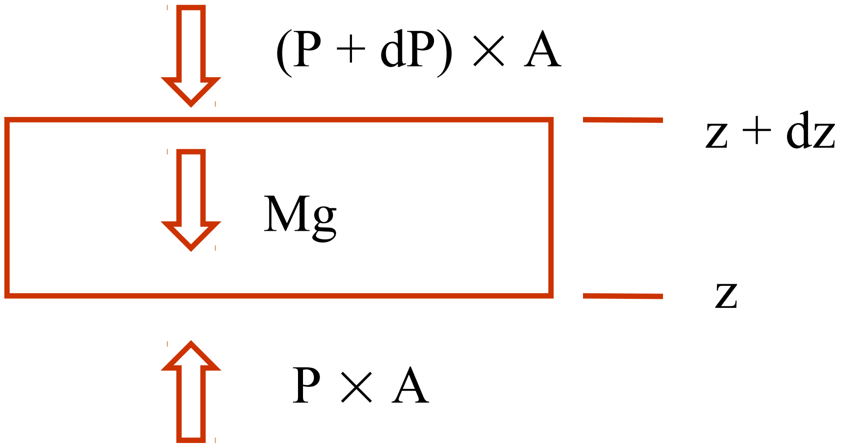
Outline for Planetary Atmospheres

- Pressure Structure
  - Hydrostatic equilibrium and “Scale Height”
- Temperature Structure
  - Adiabatic Lapse Rate
  - Radiative transfer
  - Equilibrium temperature
  - Greenhouse effect
- Sources
  - Primary – retension of solar nebula gasses
  - Secondary – outgassing from planet
  - Late arrival of volatiles during accretion
- Escape
  - Thermal (Jeans) escape
  - Photochemistry
  - Other mechanisms
- Diagnostic measurements
  - Isotopic ratios

- Second section on atmospheric dynamics (circulation)

Pressure Structure – Hydrostatic Equilibrium

- Hydrostatic equilibrium
- Same as within earth except:
  - z (altitude) is positive up
  - ρ varies <u>dramatically</u>
  - g also varies with altitude



- In equilibrium net force on any volume of air must = 0 so

$$\text{Force down} = \text{Force up}$$

$$\left(P+dP\right)\times A+Mg = P\times A$$

$$dP\times A+Mg = 0$$

$$dP=-\frac{Mg}{A}=-\frac{V\rho g}{A}=\frac{A\ dz\ \rho\ g}{A}=-\rho\ g\ dz$$

$$dP=-\rho\ g\ dz$$

- The minus means pressure drops as z (altitude) increases

- For lithosphere treat ρ as constant.
- For atmosphere solve for ρ using ideal gas law

Pressure Structure – Scale Height

The ideal gas law gives

$$P = nkT = \frac{\rho}{m} kT \quad \text{so} \quad \rho = \frac{m}{k}\frac{P}{T}$$

where $m =$ mass of molecule

and $k =$ Boltzmann's constant

so

$$dP = -\rho\ g\ dz = -\frac{m}{k}\frac{P}{T}\ g\ dz$$

$$\frac{dP}{P} = -\frac{mg}{kT} dz = -\frac{1}{H} dz \quad \text{where} \quad \boxed{H = \frac{k\ T}{m\ g} = \quad scaleheight}$$

If H is constant $\left(i.e.\, m,\, T,\, g \text{ constant}\right)$ then we can integrate to get

$$\ln\left(\frac{P}{P_0}\right) = -\frac{z}{H}$$

$$\boxed{P = P_0\, e^{-z/H}}$$

where P_0 is pressure at $z=0$

so the pressure drops by $1/e$ for every H that you ascend.

Pressure Structure – Examples

$$H=\frac{k\,T}{m\,g} \qquad P=P_0\,e^{-z/H}$$

- Over broad region H usually is approximately constant
  - T is in Kelvin – so ~300K for earth. A few 10’s of K make little difference
  - As long as z << Radius then g is approximately constant
  - As long as composition stays fixed m, is constant

$$H=\frac{k\,T}{m\,g}=\frac{1.38\times10^{-23}\ \mathrm{J\,K^{-1}}\ \times\ 300\ \mathrm{K}}{30\ \mathrm{amu}\times1.66\times10^{-27}\ \mathrm{kg/amu}\times9.8\ \mathrm{m\,s^{-2}}}=8500\ \mathrm{m}=8.5\ \mathrm{km}$$

Example: at $z=2000$ m (the approximate elevation of Laramie) the pressure is

$$P=P_0\,e^{-2000/8500}=P_0\,e^{-0.23}=P_0\,0.79=79\%\text{ of sealevel pressure}\approx0.79\text{ bar}$$

$$\text{At } z=8848\text{ m (Everest) } P=P_0\,e^{-8848/8500}=P_0\,e^{-1.11}=0.33\ P_0$$

Pressure Structure – Other Planets and High Altitudes

$$H = \frac{k\,T}{m\,g} \qquad P = P_0\,e^{-z/H}$$

- Increasing g decreases H – i.e. the atmosphere is more compressed
- Increasing T increased H – i.e. the atmosphere is more expanded

- Increasing m (mean molecular weight) decreases H – more compressed

- Applications:
  - Other terms being equal, an H_2 atmosphere with m = 2 amu
    will be 15 times as spread out as our atmosphere with m $\approx$ 30
    - If you know g and T you can estimate the composition (or at least the mean molecular weight) of the atmosphere by measuring H

  - You get a different H for each gas.
    - In the lower atmosphere turbulence keeps the gasses mixed and only the <u>mean</u> molecular weight matters.
    - In the very high atmosphere the gasses can separate and the gas with the largest H (and lowest m, so usually H_2 or H) spreads out and dominates at high altitudes

Temperature Structure (1)

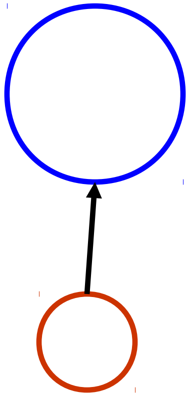
How will temperature fall with altitude?
That will be related to something called the "**adiabatic lapse rate**"

If you let a gas expand or contract adiabatically,
that is, without allowing heat in or out of the gas, then

$PV^{\gamma}=\text{constant}$ or equivalently $\left(\frac{T_1}{T_0}\right)=\left(\frac{P_1}{P_0}\right)^{\frac{\gamma-1}{\gamma}}$

where γ is a material constant equal to $\sim 4/3$ for polyatomic gasses.

What happens if you lift a parcel of air a small Δz and
let it expand adiabatically as the pressure drops?
It will cool an amount given by the equations:



$$\left(\frac{T}{T_0}\right)=\left(e^{-\frac{\Delta z}{H}}\right)^{\frac{\gamma-1}{\gamma}}=e^{-\left(\frac{\gamma-1}{\gamma}\frac{\Delta z}{H}\right)}\approx 1-\left(\frac{\gamma-1}{\gamma}\frac{\Delta z}{H}\right)$$

where we used the approximation $e^{-x}\approx 1-x$ for small x.

Temperature Structure (2)

We want to use this to calculate $\frac{\mathrm{dT}}{\mathrm{dz}}$ in convenient terms.

$$\frac{T}{T_0}=1-\frac{\gamma-1}{\gamma}\frac{\Delta Z}{H} \quad \text{or} \quad T=T_0-T_0\frac{\gamma-1}{\gamma}\frac{\Delta Z}{H} \quad \text{or}$$

$$T-T_0=-T_0\frac{\gamma-1}{\gamma}\frac{\Delta Z}{H}$$

$$\Delta T=-T_0\frac{\gamma-1}{\gamma}\frac{\Delta Z}{H} \quad \text{so in the limit of small } \Delta z$$

$$\frac{dT}{dz}=-T\frac{\gamma-1}{\gamma}\frac{1}{H}=-\frac{T}{H}\frac{\gamma-1}{\gamma}=-T\frac{mg}{kT}\frac{\gamma-1}{\gamma}=-\frac{mg}{k}\frac{\gamma-1}{\gamma}=-\left(\frac{g}{c_p}\right)$$

$$\Gamma_{\text{adiabatic}}\equiv-\frac{dT}{dz}=\frac{T}{H}\frac{\gamma-1}{\gamma}=\frac{mg}{k}\frac{\gamma-1}{\gamma} \quad \text{is called the } \textit{adiabatic lapse rate}.$$

From the last equation you can see it depends only on material properties of the gas, and g.

Temperature Structure (3)

$$\Gamma_{\text{adiabatic}} \equiv -\frac{dT}{dz} = \frac{T}{H}\frac{\gamma-1}{\gamma} = \frac{mg}{k}\frac{\gamma-1}{\gamma}$$

What kind of adiabatic lapse rate do we have for Earth?

$$\Gamma_{\text{adiabatic}} = \frac{T}{H}\frac{\gamma-1}{\gamma} = \frac{300\text{ K}}{8800\text{ m}}\frac{\gamma-1}{\gamma} = \frac{300\text{ K}}{8.8\text{ km}}\frac{\gamma-1}{\gamma} = 34\text{ K/km} \times \frac{\gamma-1}{\gamma}$$

$$= 34\text{ K/km} \times \frac{7/5-1}{7/5} = 34\text{ K/km} \times \frac{2}{7}$$

$$= 9.7\text{ K/km}$$

gas type	**γ**
monotomic	5/3
diatomic	7/5
polyatomic	4/3

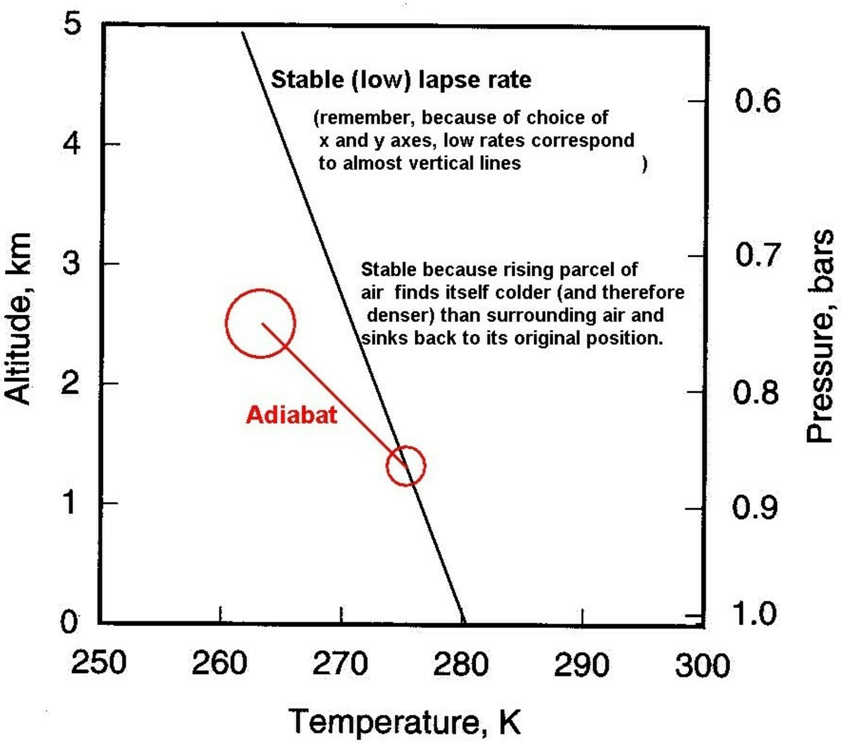
Laramie is ~ 2000 m $= 2$ km above sealevel, so from this formula it should be 2 km × 9.7 K/km = 19 K = 34 F cooler than sealevel.

This is a bit of an overestimate (for reasons we'll see later) but isn't bad for this simple of a calculation.

Note, 34 K out of 300 K is only a 10% change, which is why earlier we could use the *isothermal* equations for pressure.

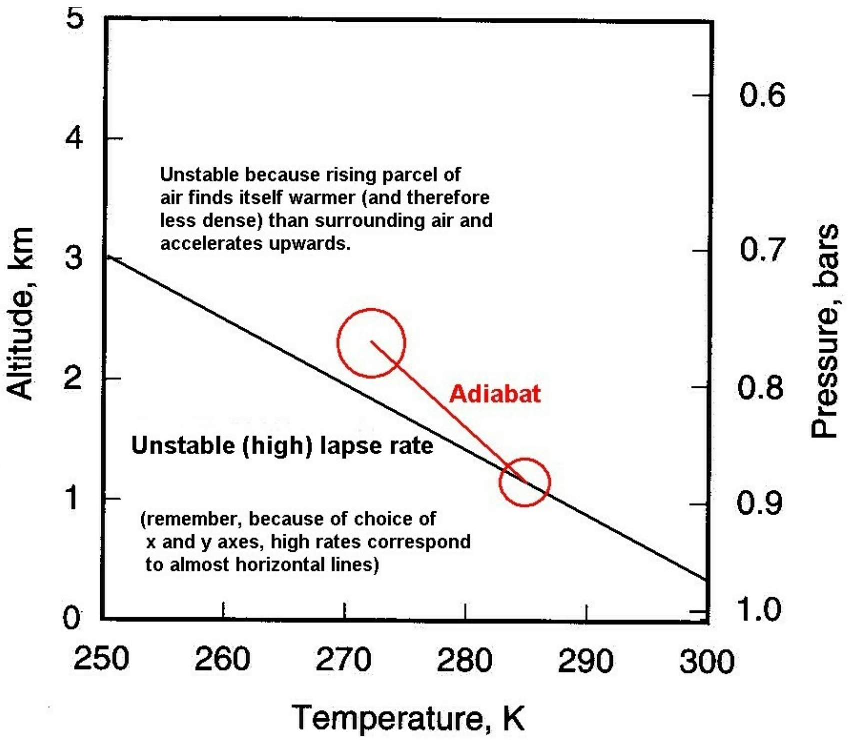
Will convection occur?

- Convective stability determined by comparing actual lapse rate Γ to theoretical adiabatic lapse rate $\Gamma_{adiabatic}$
- $\Gamma < \Gamma_{adiabatic}$ stable (no convection)
- $\Gamma > \Gamma_{adiabatic}$ unstable (convection)



(remember, because of choice of x and y axes, low rates correspond to almost vertical lines)

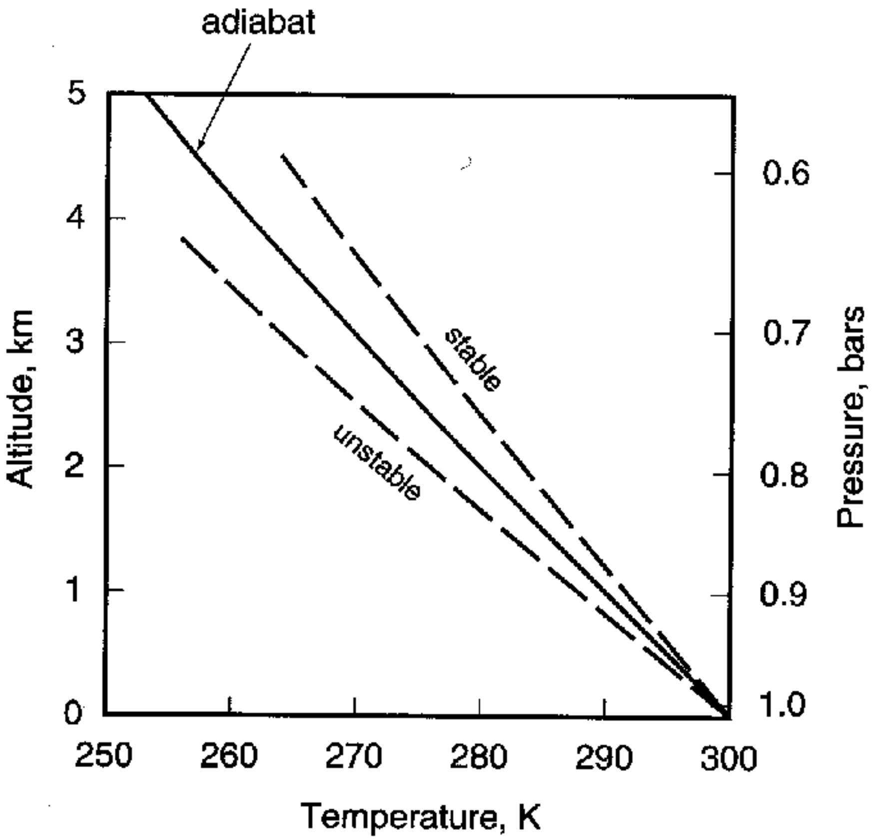
Stable because rising parcel of air finds itself colder (and therefore denser) than surrounding air and sinks back to its original position.



Unstable because rising parcel of air finds itself warmer (and therefore less dense) than surrounding air and accelerates upwards.

(remember, because of choice of x and y axes, high rates correspond to almost horizontal lines)

Why do we often find $\Gamma=\Gamma_{\text{adiabat}}$?



- Suppose you start with a stable atmosphere with low Γ
  (for example $\Gamma=0$ which means the atmosphere is isothermal)

- Add heat to the bottom of the atmosphere

- If heat has no other way to escape, then T will increase at the bottom making Γ larger and larger.

- When $\Gamma= \Gamma_{\text{adiabat}}$ convection will begin and will start removing heat

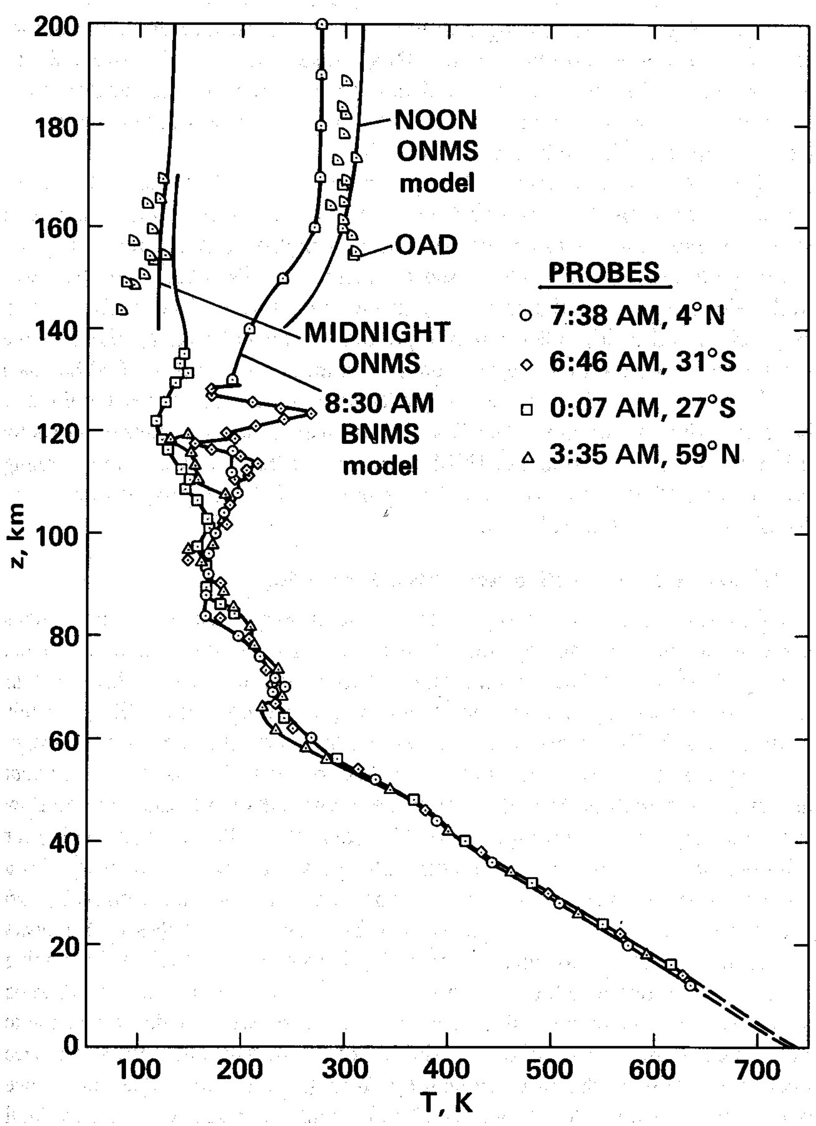
  – Convection is such an efficient process that you cannot get Γ much larger than Γ_{adiabat} before the convection is so vigorous it can remove any reasonable amount of extra energy you put in.

- If convection is required to remove heat, you have $\Gamma= \Gamma_{\text{adiabat}}$

- If some other process can remove heat fast enough to limit $\Gamma< \Gamma_{\text{adiabat}}$ then convection will not occur
  - If the atmosphere is transparent enough and the heat flux is low enough, infrared radiation can carry the energy out at low Γ.

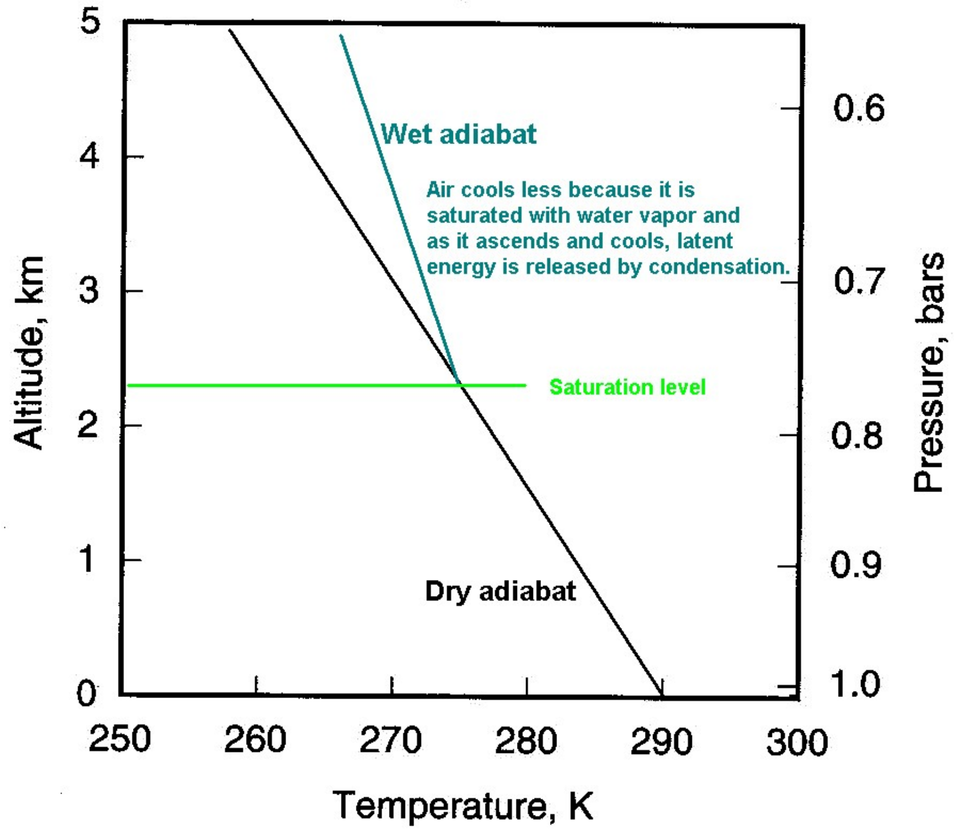
The structure of planetary atmospheres

Pioneer Venus measurements, from Seiff 1983



- If convection is required to remove heat, you have $\Gamma = \Gamma_{adiabat}$
- If some other process can remove heat fast enough to limit $\Gamma < \Gamma_{adiabat}$ then convection will not occur
  - If the atmosphere is transparent enough and the heat flux is low enough, then infrared radiation can carry the energy out at low Γ.
  - At high enough altitude low density makes atmospheres transparent enough, so Γ becomes small and convection stops.
  - That point (the tropopause) defines the boundary between the troposphere and the stratosphere.

How Water Vapor produces a “Wet Adiabatic Rate”



- Unsaturated or “Dry” $\Gamma = \Gamma_{\text{adiabat}}$
- Saturated: $\Gamma = \Gamma_{\text{wet adiabat}}$

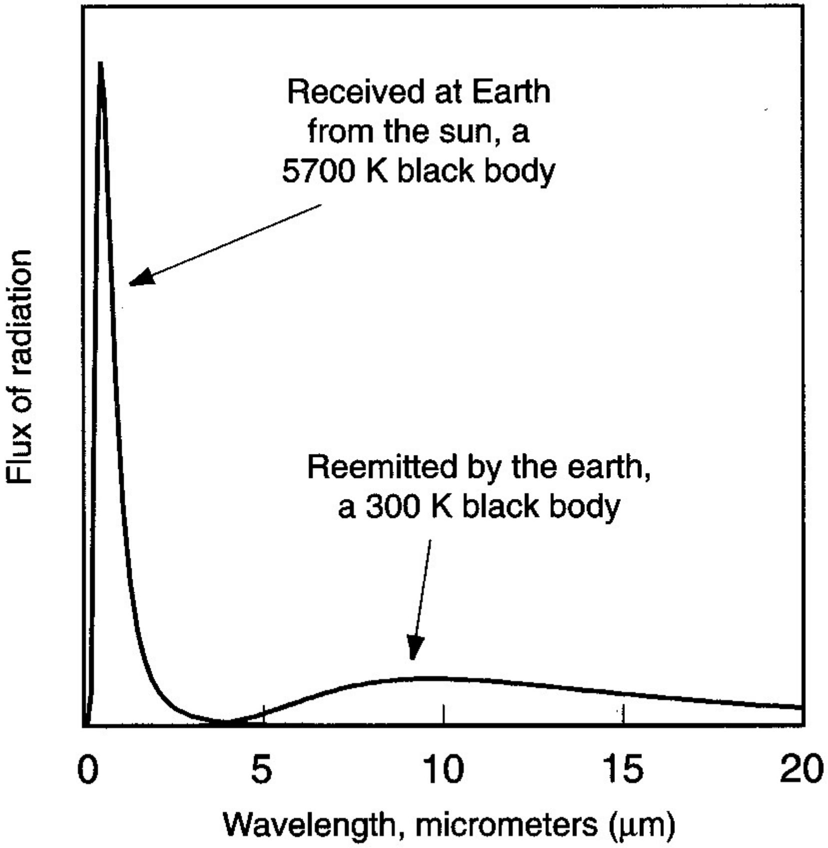
- Thunderstorms work because a saturated parcel of air cools slower as it ascends, finds itself warmer than the surrounding air, therefore continues to accelerate upward.

- In a thunderstorm the flat bottom of the cloud corresponds to the saturation level
- With a sky of many cumulus clouds, the common base is also the saturation level

- The saturation vapor pressure of H_2O drops rapidly with temperature and altitude, so not a lot of H_2O vapor can make it to high altitude. (Cold trap)

- All of these same principles applies to convection and ascent of magma within the earth, but the “equation of state” and “condensation rules” are more complicated.

The nature of the radiative balance



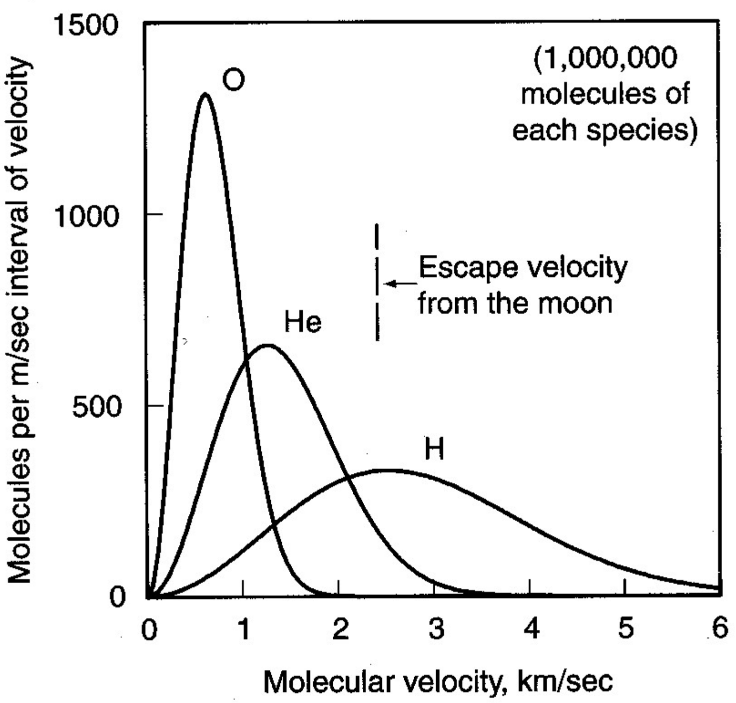
From our text by Wood Fig. 5.1

- A planet will heat up till the outgoing radiant energy from the sun balances that coming in from the sun.
  - In the curve at left it is the integral of the area under the two parts of the curve which must balance
- The outgoing energy is at a much longer wavelength because the earth is much cooler than the sun.
  - $\lambda_{max} \approx (3000\ \mu m\ K)/T$
  - For Sun λ_{max}=3000 μm K)/5700 K = 0.5 μm
  - For Earth λ_{max}=3000 μm K)/300 K = 10 μm
  - Rather than remembering the exact constant, just remember the inverse nature of the relationship and the fact that you radiate at 10 μm
- Greenhouse effect: Because the outgoing radiation is all in the infrared, anything that blocks the infrared will tend to heat up the surface of planet.

The sources of atmospheres

- Primary atmospheres
  - The H, He, etc. captured and retained from the original solar nebula
  - Jupiter, Saturn (and to some extent Uranus and Neptune) have primary atmospheres
- Secondary atmospheres
  - Later outgassing from the solid material of the planet
  - Later accretion from outer solar-system planetesimals and comets

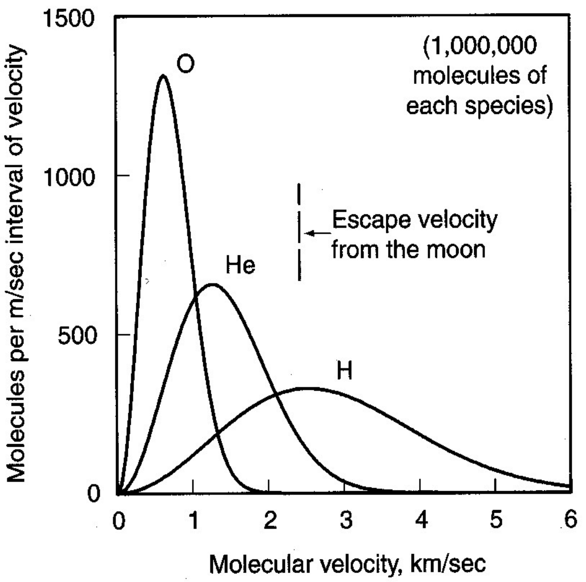
Thermal Escape of Atmospheres



- If a molecule (or atom) at the top of the atmosphere has v > v_{escape} it can leave the planet
- Typical velocity $v_f = \sqrt{\dfrac{2kT}{m}}$

  where $m =$ mass of molecule or atom
- Because some molecules are going significantly faster than the average, you need 10 $v_v < v_{escape}$ to retain an atmosphere over the 4.5 billion year lifetime of the solar system.
- From the plot at left, the moon will lose all its H almost immediately, but will also lose its He, O, etc. over a geologically rather short time.
- Because v_{escape} is greater for the Earth, it can retain its O (and most He) but any H <u>which makes it to the top of the atmosphere</u> will be lost.

Variables in Thermal Escape



Test is $10\,v_f < v_{escape}$

or since $v_{escape} = \sqrt{\frac{2GM}{R}}$

where M is the mass of the planet
and R is its radius.

the test for retention becomes

$$10\sqrt{\frac{2kT}{m}} < \sqrt{\frac{2GM}{R}}$$

- General rules
  - Heavy gasses will be retained better than light ones
  - Colder planets will retain an atmosphere better than warm ones
  - Larger planets will retain an atmosphere better than small ones

- Complications
  - Ultraviolet light can break apart heavy molecules into light atoms at the top of the atmosphere, so you need to work out velocities for atoms, not just molecules
    - H and O, not just H_2O
  - Thermal escape only works if the molecule can make it to the top of the atomosphere. (Otherwise it will collide with another atom before escape.) If the molecule in question is something that will freeze out as solid or liquid before it gets very high, then it will not escape.
    - This "cold trap" preserves the H_2O for the earth.

Non-thermal escape

- Chemical reactions (photochemistry) can produce high velocities
  - If solar ultraviolet photons ionize CO_2 then the recombination reaction $CO_2^+ + e^- \rightarrow CO + O$ breaks apart the molecule and produces a very high speed O

- Solar wind sweeping
  - The high speed (500 km/s) tenuous wind from the sun can sweep away atoms
    - Planets with strong magnetic fields are shielded from this because the ionized wind is deflected before hitting the planet

- Giant impacts
  - A large enough impact ejects not only part of the solid planet, but all of the atmosphere within a light of sight of the blast.

- Hydrodynamic escape
  - If high temperatures in the upper atmosphere produce $v_{thermal}$ even close to v_{escape} then there is a rapid flow of material away from the planet
- Storage in the planet
  - Chemical reactions could remove material from the atmosphere and transform it to solid form
    - Limestone on the earth stores what may have originally been CO_2 from the atmosphere

Diagnostic tests for past escape

- Heavy isotopes will have slower velocities (and therefore slower escape rates) compared to light isotopes

- The atmosphere will become enriched in heavy isotopes of an escaping species
  - The effect is biggest for ^{2}H (=D) vs. ^{1}H because of the $\sqrt{2}$ difference in V.

    - For Venus D/H ~ 2×10^{-2}, which is ~150 times larger than on Earth
      - Venus has lost most of its H (from H_2O)

    - For Mars D/H ~ 8×10^{-4}, which is ~5 times larger than on Earth
      - Mars has lost much of its H (from H_2O)

    - Complications
      - Relative loss rates of D, H uncertain because of nonthermal escape
      - Original volatiles (from hydrated asteroids, comets) have somewhat different D/H

  - Because noble gasses like He, Ne, Ar, Kr, Xe are concentrated in the atmosphere, they also provide good diagnostics.