Planetary Geology 4460 Homework SOLUTION #4 Due Sept. 29, 2017

Problem 1. (10 points) Impact Cratering Energies Assume an asteroid with a diameter of 5 km and a density 3.5 gm cm^{-3} impacts the earth, with a velocity of 20 km s^{-1}. Find the kinetic energy in Joules which will be released.

Kinetic energy is $\frac{1}{2}mv^2$ *and in this case* $m = \rho\ 4/3\ \pi\ r^3$. *Converting to SI units before calculating, this will be* $3500\ kg/m^3 \times 4/3\ \pi\ (2500\ m)^3 = 3500\ kg/m^3 \times 6.54 \times 10^{10}\ m^3 = 2.29 \times 10^{14}\ kg$. *With this mass and* $v = 20 \times 10^3$ *m/s the kinetic energy is* $\frac{1}{2}mv^2 = 4.59 \times 10^{22}\ km\ m^2 s^2 = 4.59 \times 10^{22}$ *joules.*

Problem 2. (5 points) Olivine melting A Suppose that you have solid olivine with a composition of 50% Forsterite and 50% Fayalite. You heat it till the first melt begins to appear, then let that small amount of melt escape to the surface. At what temperature does that melt first form? What is the composition of that melt? Use the olivine phase diagram from Winter on the following page.

A vertical line at 50% composition intersects the solidus at a temperature of 1440K. Moving horizontally till you find the intersection of 1440K with the liquidus gives a composition of 18% Fosterite and 82% Fayalite.

Problem 3. (10 points) Olivine melting B Suppose that you again have the 50% Forsterite 50% Fayalite solid but this time you heat it till approximately 20% has melted. At what temperature has 20% melted? What is the composition of that melt?

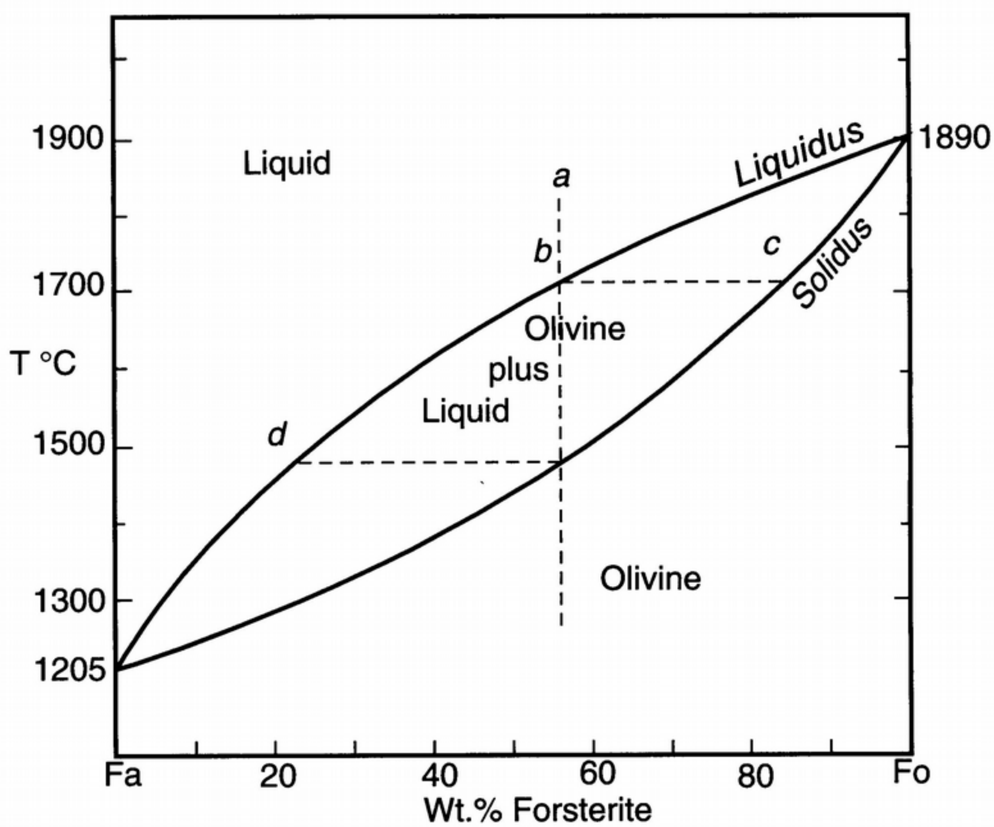
By the lever rule
$C_{SYSTEM} = F_{MELT} \times C_{MELT} + F_{CRYSTAL} \times C_{CRYSTAL}$
Since $F_{MELT} + F_{CRYSTAL} = 1$ *this can be rewritten as*
$C_{SYSTEM} = F_{MELT} \times C_{MELT} + (1-F_{MELT}) \times C_{CRYSTAL}$
$(F_{MELT} + F_{CRYSTAL}) \times C_{SYSTEM} = F_{MELT} \times C_{MELT} + (1-F_{MELT}) \times C_{CRYSTAL}$
Numerically you could try various Temperatures (and the corresponding pairs of C_{MELT} *and* $C_{CRYSTAL}$ *values until you satisfied the above equation.*

However it is probably easier to find this graphically using the lever rule. That is the mathematical equivalent of rearranging the equation as follows. Again since $F_{MELT} + F_{CRYSTAL} = 1$ *we can modify the first equation to read*
$(F_{MELT} + F_{CRYSTAL}) \times C_{SYSTEM} = F_{MELT} \times C_{MELT} + F_{CRYSTAL} \times C_{CRYSTAL}$ *then collecting F terms*
$F_{MELT} \times (C_{SYSTEM} - C_{MELT}) = F_{CRYSTAL} \times (C_{CRYSTAL} - C_{SYSTEM})$

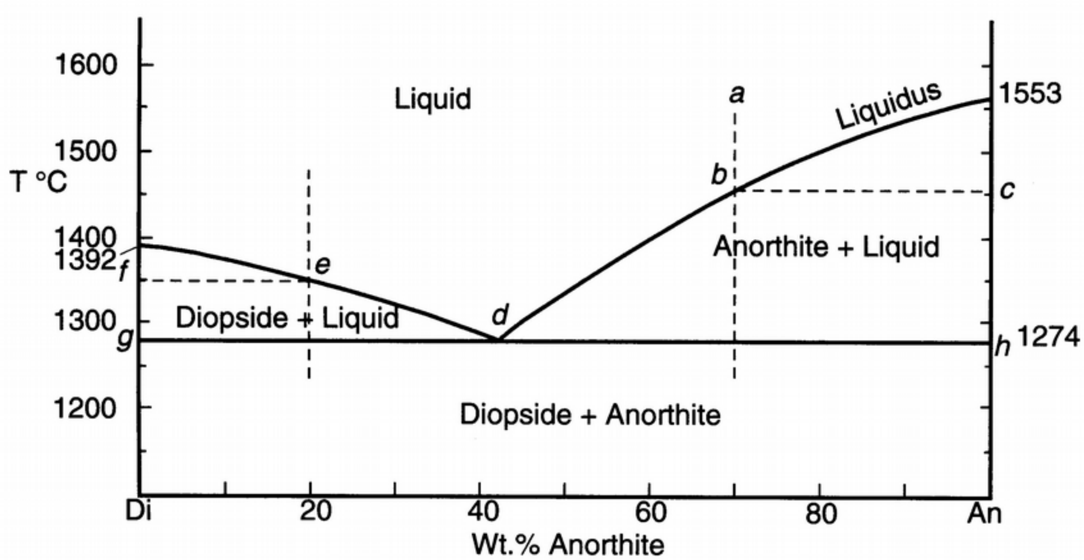
Define C to be 1 for pure Forsterite and 0 for pure Fayalite.
In this case $C_{SYSTEM} = 0.5$ *and* $F_{MELT} = 0.2$ $F_{CRYSTAL} = 0.8$ *so*
$F_{MELT} \times (C_{SYSTEM} - C_{MELT}) = F_{CRYSTAL} \times (C_{CRYSTAL} - C_{SYSTEM})$ *becomes*
$0.2 \times (0.5 - C_{MELT}) = 0.8 \times (C_{CRYSTAL} - 0.5)$ *or*
$(0.5 - C_{MELT}) = 4 \times (C_{CRYSTAL} - 0.5)$
We draw a vertical line at $C_{SYSTEM} = 0.5$ *and then using a ruler we look for the spot where the horizontal distance from that vertical line to the liquidus is 4 times the horizontal distance from that vertical line to the solidus. With a cm ruler I placed the 10 on the vertical line and after a few trials found that I could make the factor of 4 hold when the left distance was 2.70 cm and the right distance was 0.675 cm. The exact distances you get may be different, depending on how the xerox machine scaled the plots. However regardless of the scaling of the plot this should happen at a temperature of approximately 1484º C very close to point "d". The composition of the melt is then 22% Forsterite and 78% Fayalite.*

Problem 4. (5 points) Diopside Anorthite Crystallization. Suppose you have a melt which is 70% Anorthite and 30% Diopside, and it cools till crystals begin to form. You continue cooling it till a very small amount of melt is left, that then escapes and erupts onto the surface. What is the composition of that final melt, and what temperature does that occur at?

The composition and temperature (assuming neither changes after the magma escapes) must the the <u>*eutectic*</u> *composition and temperature which is 42% Anorthite, 58% diopside, at a temperature of 1274º C.*



1900
1700
T °C
1500
1300
1205
1890
Liquid
Liquidus
Solidus
a
b
c
d
Olivine
plus
Liquid
Olivine
Fa
20
40
60
80
Fo
Wt.% Forsterite



1600
1500
T °C
1400
1392
1300
1200
1553
1274
Liquid
Liquidus
a
b
c
d
e
f
g
h
Anorthite + Liquid
Diopside + Liquid
Diopside + Anorthite
Di
20
40
60
80
An
Wt.% Anorthite