Homework #5 earth surface processes Humphrey 2016,

There are 2 questions. Everybody should get exactly the same results, since all data is included. Question 1 is a repeat question to give you a chance to do a question that most of you did poorly, and which might appear on the midterm (hint!). If you did it correctly before, it will only require copying just the answers (and checking), but for some of you it will be a second chance. Question 2 will be discussed in more detail in class on Tuesday, although there is enough here for you to start thinking about it. Question 2 will require some work and some thinking!

1 Please redo the question from homework #3 on a shallow soiled hillslope, with a soil depth of 2m (slope perpendicular depth) over solid bedrock. The slope is uniform, and the soil is essentially a slab lying on a uniform tilted slope of bedrock. The slope is 20 degrees, and the internal angle of friction of the soil is about 30 degrees. The soil is homogeneous and the bedrock has very low conductivity. The soil has a hydraulic conductivity of 10^{-6}m/s, a porosity of .3, a density of 1600 kg/m^3 and the soil has no cohesion. It is raining very hard and the soil is saturated with a water table 0.5m below surface.

Calculate:
a) the water **pressure** at the soil/bedrock interface [remember that the water is moving, since it is on a slope]
b) the driving stress (**shear stress**) at the soil/bedrock interface (you need to add the water weight to the soil weight),
c) the **effective normal stress** across the soil/bedrock surface, this is the stress which will produce friction
d) Calculate the **frictional resistance** as a stress.
e) Will the slope fail (**factor of safety**)?.
f) Would the slope fail if it were dry?

2 Hillslopes as conveyor belts, or as convolution integrals.

This homework may be very difficult for some of you, since it introduces the use of computer algorithms. If you are relatively new to computing, then do the "conveyor belt" solution in Excel outlined below. The more confident might want to try the convolution approach, best in Matlab or Python. All of you should read this entire blurb. Unlike some of the homework problems, all the information you need is included in this question, and the answer method is [hopefully] clearly pointed out. Please tell me of errors in this.

1 A purpose of this week's homework is to make sure you can all do minimal calculations and plotting in some computing language such as EXCEL or MATLAB (or the program of your choice, such as Python). I realize that many of you will use Excel, which is fine. Excel is a good program for simple calculations. You may find as you progress in your career that you may want to graduate up to some of the more powerful

programs out there. MATLAB or Python are particularly useful programs for the Geosciences. MATLAB is available in the student computer labs, and Python is similar to MATLAB but free(!) for everyone.
2 The second purpose of this exercise is to explore a particularly useful concept in much of geomorphology, particularly where there is a need to route a *distributed* input that is focused into a *concentrated* or *point* output. Technically, what we are doing is a *convolution*: that is a type of multiplication over time of two descriptors; in this case the time description of the rainfall, and the time description of the delays in the hillslope transport. The concept of a convolution of the inputs to get the outputs should be in your mind whenever you think of drainage basins, and their inputs and outputs.

The #2 Homework Question: Assume a straight uniform hillslope, with an angle of 12 degrees. The hillslope has a uniform layer of soil over impermeable bedrock. The length of the slope is 50m from the crest of the hill, down to our point of interest, which is the outlet, the stream at the toe. The x coordinate of the hillcrest is 0 and the stream is at $x=X$. The saturated conductivity of the soil is 3.4×10^{-3} m s^{-1} (we make it very permeable to make the water flow quickly), and the thickness of the slope soil is 1m, above impermeable bedrock. The porosity is 50% (higher than typical soils). The entire slope is initially at field capacity with no saturated zone, and at 1 hour *past* noon it starts to rain hard (10^{-5} m s^{-1}), it rains steadily for 4 hours, stops for 1 hour, and then rains twice as hard (2×10^{-5} m s^{-1}) again for 3 hours, stops for 2 hours, and with a final burst it rains at 3×10^{-5} m s^{-1} for one last hour. Assume (to make this problem easier) that there is a constant time delay for rain to traverse the unsaturated zone, from surface to water table. (This delay is about the time it takes for water traverse a ‘bin’ on the hillslope.)

- The problem: find the water discharge per unit width of slope at the toe of the slope as a function of time. To do this, plot discharge each hour at X =50m, over time; starting at 12 noon and going until 12 noon the next day.

You will find it necessary to answer the following questions:

- What is the water velocity in the saturated zone? Darcy’s Law, note that specific discharge [little ‘q’], has dimensions of velocity and is a reasonable scale velocity for the water movement. However, as we discussed in class, a better velocity to use in this problem is the effective velocity, which is the previous scale velocity divided by the porosity.
- How long does it take for water from the highest point on the slope to reach the “outlet”?

(Big hint: the answers are, 5m per hour, 10 hours)

Solutions:

There are three ways of doing this problem. One is to use the conveyor belt analogy; another is to code the numerical convolution approach which is discussed below, and finally you can do an analytic convolution. Unless you feel ‘computer and math savy’ I recommend for a first attempt to use the conveyor belt approach, which is most intuitive.

Interestingly, after you figure it out, the convolution approach is identical to the conveyor belt code.

Conveyor belt analogy

Since the water speed down the hillslope is constant, the hillslope acts like a conveyor belt that is moving parcels of water down the slope. You can imagine the water as sitting, stationary, on the belt, while the belt moves at 5m per hour. As the belt moves down the slope, more water is added, since it keeps raining. If you think of the water as sitting in bins on the belt, you can ask the question: How deep does the water in the bins get, by the time they reach X.

It is easy to make Matlab or Excel act like a conveyor. A big thing to notice is that the time it takes for water to move down the slope [the conveyor belt speed] is 5m per hour. So we can simplify our conveyor belt by making the bins on the hillslope 5m long (so you need 10 of them to make a 50m long slope), and by taking 1 hour time steps. That way a bin's worth of water moves downslope to the next bin location every hour. Note this just makes the calculations easier, we could use other bin sizes, but would have to take time steps of the length needed for the water to traverse a bin.

So make a column in Excel of 12 zeros (a vector in MATLAB). Each cell or zero represents a 5m wide bin on the hillslope, and the zero is the current amount of water in the bin. Bins 1 and 12 are special. Thus, the bins 2-11 represents the hillslope before the start of the rain, (12 noon), plus one extra bin to represent the stream and to catch the exit water (the 12th bin), and one at the top to represent the rainfall rate. Each bin represents 5m of hillslope and is going to be the place we put water.

Now apply this algorithm: each hour (or each model timestep) when it rains, apply (or add) 1 hour of rain-fall into each bin (that is input=$\Delta x * R(t) * 3600$), the 3600 is the number of secs in an hour. After adding the water to the bin, shift the contents of every bin down 1 row. The shifting process represents 1 hour of time occurring and the water moving 5m downslope. Record the amount of water in bin 12 (this is the OUTPUT for that time period) and the time (of the step), and then empty bin 12 by setting bin 12 to zero. Repeat, while keeping track of the model time, (and remembering to adjust the rainfall rate with time) until the rain has stopped **<u>and</u>** all the bins are empty. (this is the algorithm, see below on how to do this logically in Excel)

Your result is the amount of water in bin 12 through time, starting at 12noon and going until everything is back to zero. The amount of water in bin 12 is the water flux *q*. The only difficulty is recognizing what *q* represents. The flow into bin 12 is the flux per hour. Therefore we need to divide q by 3600 to get the flux per sec (since K_{sat} is in seconds).

Now plot *q* versus time**, label the axes with both labels, and numerical values,** and you are done! When plotting the results in Excel, note that you can 'right-click' on any of the features of the plot, such as the axes or the wording or the lines and change their properties such as font size or line styles. This can really help the plot become more readable. (please turn off the automatic smoothing, or 'spline', in EXCEL, it ruins the plots) In matlab, you can open the 'figure editor' and do the same.

A note on EXCEL: Some hints on doing the conveyor approach in Excel.

Make a spread sheet of 12 rows and 24 columns. Each column of cells represents the bins, and the amount and location of the water on the slope at the end of consecutive hours. Row 1 and 12 are special. Row 1 is reserved for the rainfall. Enter the hourly rainfall (times delta x and 3600secs) manually in the first 11 of the 24 hour columns; the last 13 columns of row 1 will be zero. The 12th row is the output. Write your formula in cell B3: B3 = A2 + A$1. Then 'fill down' and 'fill right' (in 'edit' menu). Note the absolute addressing of the A1 cell row. The output cell should be filled with B12 = A11. In effect you are implementing the algorithm from above, but you are keeping the intermediate results. Marching across the spreadsheet is marching in time, rows going down are going down the slope in space.

Cells in Excel are addressed absolutely with their names and a dollar sign, eg A11 which means the actual cell at column A row 11. More usually you refer to cells *relatively* by their column and row address, eg A11. Using relative addresses means that you can copy and paste (or *fill*) formulas and Excel can (will) change the addresses to match the location you put the formula into [eg. If you copy A11 into column C cell 11, it will appear as C11]. The absolute address of the A$1 cell means that Excel will also add from the A or B or C column, but only from the 1st cell of the column, (Excel does relative addressing by default). An address such as A$1 means that excel can change the A column to the B or C or etc as you 'fill' the spreadsheet, but it will always point to the top entry (row 1) of that column.

A note on Matlab or Python: This is a 2 or 3 line program in Matlab. Enter the rain (R(t) as an array, and then just repeatedly shift and add and plot, all at the same time.

Tutorials on Excel and MATLAB: There are numerous tutorials out there, in book form or on the web. They may help you in the above problem. Two that I have found to be useful are for Excel and MATLAB. (note these are web pages at other Universities, so you get whatever is there). A super short tutorial on Matlab from me is here. The best way to learn either is to get a brief tutorial from one of your peers. Try to find someone to show you the basics, once you have a minimal knowledge, most students can advance on their own.

For the Braver, the convolution approach: (worth reading but you do not need to do the problem this way… [you do get the same answer]):

The basic equation is:

$$q(X,t) = \Delta x * \sum_{x=0}^{X} R\left(t - \frac{(X-x)}{v_w}\right),$$ this is the equation I developed in class, but written to show that we are summing the water downslope

Which gives the water flux at time t, and where the water velocity is just K_{sat} * sin α / η and *X* is the distance down slope from the crest to the point of interest. The summation is actually over time. This can be reduced to the equation from class by making the bin size a constant. This becomes even easier if we define the length of the bins (Δx) to be the distance water flows in 1 hour, i.e. $\Delta x = v_w * 3600$ secs/hr, or 5m. Finally we recognize that the time the water takes to travel across each bin is Δt, which is constant. Replacing the sum in space coordinates with a sum backwards in time, the equation looks like: (this was the last equation from class)

$$q(X,t) = \Delta x * \sum_{j=0}^{j=T-1} R(t-(j+1)*\Delta t)$$

, where Δt is one hour, and 't' and therefore 'j' are also in 1 hour steps, the variable T in the summation limit is the time in hours for water to flow from the crest to X: T=10.

You can set it up as a spreadsheet or a MATLAB problem, and do the summation. For this problem include either your Excel formula, or your MATLAB or whatever language code with your homework. Plot the resulting water discharge versus time, and you are done!

For the Analytically inclined:

If you have learned convolutions in a math class, you can do this problem as a convolution of 2 functions: namely the input (the rainfall R(t)) with the transfer function of the hillslope. The transfer function in this case is a boxcar function (constant amplitude and length of 10 hours), which makes the analytic convolution very simple. As a hint, you can add the 3 convolutions, 1 for each rainfall period. That way the output is just the sum of 3 boxcar to boxcar convolutions.