Homework #9 GEOL 4880 Humphrey Fall 2016

These are not due until AFTER thanksgiving break. The first 2 questions are basic to river studies. The 3rd question is a little tricky, but actually fairly easy. The 4th question looks complex, but is just solving a cubic equation. Don't panic, there are cubic equation solver apps out there in web-land. There are a lot of calculations in this homework, take your time and think a little. And underline or box your answers!

1) **a**. Estimate the Hydraulic Radius of the Laramie River at flood, 1m deep and 4m wide. Assume a trapezoidal channel shape, 4m wide at the surface, with straight 45 degree sloping banks, so that the 'thalwg' or middle (flat) section is 2m wide

   b. Search the web for an appropriate value of Mannings 'n' for the Laramie river. You will have to look at pictures of other rivers and find one that looks similar. The USGS has a good web site for this. Calculate the Manning velocity of the Laramie river in flood. For the Laramie river in flood use (use these values for all questions on the Laramie in this homework) a depth of 1m, flow velocity of 1m/s, slope of 2×10^{-4}.

   c. We talked about the logarithmic velocity profile that develops above a rough bed. For rivers such as the Laramie river we could use: v(average) = 2.5 v* log(Depth/D_{84}), where the D_{84} can be considered the size of the roughness particles on the bed (pea gravel), but is technically the 2 sigma size of the coarse tail of sediment distribution on the bed. Note, particle roughness and depth have to measured in the same units, and that the velocity is in m/sec. Apply this to the Laramie river in flood, and compare with the velocity obtained from Mannings equation.

   d High Froude number flow (super critical or shooting flow) is relatively rare and usually only found in steep bedrock rivers. Calculate the velocity that would be needed in the Laramie river to achieve super critical flow; depth and slope being constant.

   e (quite hard, mini puzzle) Assuming the discharge and the roughness stay the same, but allowing the slope and depth to vary, how steep would the Laramie river have to be to reach a Fr of 1? (use Mannings). Keep the discharge at 3m^3/s.

2) I talked in class about energy in river flow, and showed that the Froude number can be interpreted as the ratio of kinetic energy to potential energy in the flow. Many questions in river flow can be addressed by examining the total energy of the flow. Energy, per unit width and unit length, (in other words a square meter column of water) of the flow can be expressed as a height of water (similar to the concept of hydraulic head in Darcy's groundwater flow). To be precise the energy in a column is equal to the potential energy above the bed (ρgh) plus the kinetic energy ($\rho v^2/2$), 'h' is the

average depth (depth/2), and ‘v’ is the velocity of the water. Dividing this by density*gravity turns this into the energy head of the flow: $E = h/2 + v^2/(2g)$ [note: engineers tend to use $E = h + v^2/(2g)$]

In most river flows, by far the largest part of this energy head is just the depth of flow, with the kinetic energy only a few extra centimeters of head. Because of this we often ignore the kinetic energy.

To get a sense for the amount of energy in river flow, we calculate several energies (expressed as water head) Calculate:

a. the potential energy per width, per meter length of the Laramie river (as a head, relative to **local elevation**).

b. the kinetic energy of the flow (as a **head**)

c. the potential energy lost by a column of water per meter of flow down river (as a head change)

d. the Froude number for the Laramie river

e. what super elevation is expected on a bend in the Laramie river (assume the bend has a radius of 20m and the width is about 4 meters).

3) While we are talking about energy: energy is expended by river water to move its load of sediment. This sort of calculation is very difficult to do correctly, but we can approximate the energy by saying that the water flow has to counteract the settling velocity. We will try this for the Laramie River (flow parameters in previous question). We will make several assumptions: the only sediment in transport is sand (0.2mm), and the amount in transport (mass q_{sed}) is 0.1kilograms/(m^3*s). Remember, energy is Force * distance. Force is easy; it is the weight of the sediment, distance is less obvious, but think of the settling velocity of the sand.

Compare the energy to move the sediment with the energy of the flow from ‘2c’ above. (You will need to calculate the energies in the same units!)

4) In class I quickly sketched the energy argument for the difference in behavior of super-critical and sub-critical flow. The development in class was fairly fast, but I want you to understand this important concept: that water only has so much energy, and the trade off from kinetic to potential (or vice versa) controls a lot of the rivers behavior. I would like you to follow the logic outlined here and find solutions to illustrate this behavior.

Consider a rectangular channel of constant width and steady discharge, which has a small bump of height ‘**h**’ which the flow must flow over. Assume the slope of the flow is so low that we can ignore it and set the elevation of the bed of the incoming flow to be 0, while the bed of the flow over the bump is just **h.** We will call the initial location **1** and the location at the bump **2**.

The incoming depth is $\mathbf{d_1}$ and the incoming velocity is $\mathbf{v_1}$, since the width is constant, we just look at a 1m wide section of this flow. The flow over the bump will have depth $\mathbf{d_2}$ and velocity $\mathbf{v_2}$, and the water surface height over the bump is $\mathbf{d_2 + h}$. Although we don't yet know what $\mathbf{d_2}$ or $\mathbf{v_2}$ is: that will tell us if the water surface goes up or down, and that is what we solve will for. The mass flux of water per time incoming is $\rho * v_1 * d_1$ and the mass over the bump is the same (or equivalently $\rho * v_2 * d_2$).

The kinetic energy of the water is the mass times the square of the velocity divided by 2, while the potential energy is the mass times gravity times the height ($\mathbf{d_1}$/2). To get the total energy in a column we must multiply by the depth, and then also we need to multiply by the velocity, to get the energy flux per unit width per time. We assume no energy is lost between points 1 and 2, and so we can write the energy balance for K.E. and P.E. as a statement that the energy flux per time at the two places is equal: **[equation A]**:

$$\rho * v_1^3 * d_1/2 + \rho * g * v_1 * d_1 * d_1/2 \;=\; \rho * v_2^3 * d_2/2 + \rho * g * v_2 * d_2 * (d_2/2 + h) \;,$$

where we have been careful to include the extra height of the bump (**h**) in the potential energy at location **2**. This simplifies considerably since $\rho * v_1 * d_1$ is equal to $\rho * v_2 * d_2$. Further simplification occurs if we write $\mathbf{v_2 = v_1 * d_1/d_2}$ and multiply the whole equation by $\mathbf{d_2^2}$, to get **[equation B]:**

$$0 = [g * v_1 * d_1] * d_2^3 + [2 * g * d_1 * v_1 * h - v_1^3 * d_1 - g * v_1 * d_1^2] * d_2^2 + [v_1^3 * d_1^3],$$

where all terms in '[ ]' brackets are known if we know $\mathbf{v_1 * d_1}$. This is a cubic equation in $\mathbf{d_2}$. (e.g. the equation is simply $0 = a*d^3 + b*d^2 + 0*d + c$)

a. Write equation A, and produce equation B by the indicated steps.

b. To see what the Laramie river might do in encountering a bump in the bed, try putting an input velocity of 1m/s, a depth of 1m, and a bump height of 0.1m. How much does the water surface drop over the bump? (don't forget to add the height of the bump to get the water surface. You can directly solve the cubic, or use a web app. [you are looking for the real root] Or it is a fairly quick iteration to get a solution: try a d_2 of 0.6m to start, improve on that if you can)

c. If you have made it this far; Finally, just to see the opposite behavior for high F_r number flow, try this with a flow velocity of 4m/s, depth of 1m and bump of 0.1m.