Problem #2: A first look at steady state **Due 9/22/2017**

In this course, we will be looking at *feedbacks* within various Earth systems. A key concept in all of this is ***steady state***. The calculations here are intended to familiarize you with this concept, which applies to many other systems besides the water in a lake! This might include gasses in the atmosphere, or chemicals dissolved in water – or even the amount of ice in Greenland.

If you have not already heard of "residence time", it is equal to the ***size of a reservoir of material*** divided by the ***rate at which material enters and leaves the reservoir*** at steady state. If an auditorium contains 1000 people, and people are entering and leaving the auditorium at a rate of 10 people per minute, the "residence time" of people in the auditorium is (1000 people)/(10 people/minute) = 100 minutes.

1) In the auditorium example above, imagine that we can stop people from coming in. How long will it take, given an outflow of 10 people per minute, for the auditorium to be empty?

NOTE: The "residence time" is therefore equal to the time it would take for the reservoir to empty if inputs were turned off, and it is the time it would take for the reservoir to fill if the outflows are turned off.

IN THE NEXT QUESTIONS, we slowly introduce feedback into the system:

2) A small lake contains 100 acre-feet of water (one acre-foot is the amount of water it takes to cover an area of one acre 1 foot deep in water). Water flows into the lake via a stream that carries 4 acre feet of water per day, and water flows out of the lake at the same rate. What is the residence time of water in the lake?

3) Assume that the lake in problem #2 has an outflow rate depends on the height of the water. The higher the water level, the faster water flows out of the lake (this is a source of feedback). We write this down by saying that the rate of outflow is ***proportional*** to water height, so that:

Outflow = **[**2 acre-feet per day per foot of water height**]** x **[**water height in feet**]**

The "2 acre-feet day per foot of water height" is a **constant** characteristic of this particular lake. It says that if water height is 1 foot, outflow will be 2 acre-feet per day, and that if water height is 2 feet, outflow will be 4 acre-feet per day, etc. Of course, outflow will be zero if the water level is at or below the height of the streambed leaving the lake, so all heights in feet are relative to the outlet streambed height (if the lake is deeper than the outlet streambed, as is usually the case, there can still be water in the lake even with no inflow and no outflow… we are also ignoring evaporation which makes things a bit more complicated…). REMEMBER: *At steady state, the rate of outflow is equal to the rate of inflow.*

a) Calculate the height of the water in feet from #2. **Hint:** assume that lake is at *steady state*; that is, **that inflow and outflow are equal**. All you have to do is mathematically equate the inflow with the outflow and then solve the resulting equation for water height. Express this both as a number for the particular inflow (from problem #2) and as a general equation that would be useful if inflow changes:

b) Let us assume that inflow suddenly rises to 6 acre-feet per day. What will the new steady state water height be?

c) Let's say that the inflow rate reaches up to 9 acre-feet per day in the late spring (i.e., this is its highest "normal" value), and reaches as low as 1 acre foot per day in winter. What are the corresponding "normal" high and low water levels at steady state?

d) Let's now say that you own a lake-front trophy house that will get flooded if the water level exceeds 6 feet. What would inflow need to be in order to flood your house?

4) Soon, we will learn about Earth's energy budget. At Earth's distance from the Sun, we get about 1,366 Watts of power (Joules of energy per second) for every square meter (assuming we're out in space and our square meter is perpendicular to the direction from the Sun). Earth reflects away about 30% of this power flux, and because the Earth is a sphere (not every square meter is perpendicular to the sun), this comes to only 239 Watts per square meter (W/m^2) averaged over Earth's area and averaged over time (i.e., sometimes it's night and input is zero, and sometimes it's day and input can be much higher than 239 W/m^2). So 239 W/m^2 is the Earth's average power input. If there were no energy *outputs*, Earth's temperature would keep rising very rapidly just as the water level in the lake would keep rising if there were no water outputs. The Earth's total area is 5.1×10^8 km^2.

a) Calculate the total energy input to planet Earth from the Sun. Compare this number to the total power generated and used by human beings on Earth, which is currently about 16 terawatts (TW), 16×10^{12} Watts:

b) The Earth keeps very close to energetic steady state. That is, the energy output is almost equal to energy input. Earth keeps cool by radiating energy. As we shall see, objects radiate energy according to a simple law related to the temperature of the object: radiant energy = σT^4 where σ is called the "Stefan-Boltzmann constant" equal to 5.678×10^{-8} Watts/m^2/Kelvin^4 and T is temperature in Kelvin (the Kelvin scale is the same as the Celsius scale, except that instead of 0 being at the freezing point of water 0 is at absolute zero and the freezing point of water is 273K). Write an equation that expresses Earth's power budget as a steady state (input equals output):

c) Solve your equation in part (b) for temperature in Kelvin. According to this calculation, what should Earth's temperature be (in Kelvin and Celsius)? Earth's surface has an average temperature of 15°C. How much warmer is Earth than it "should" be according to the steady-state calculation?

d) The steady-state calculation above assumes that the Earth is an ideal black-body radiator of energy, that is, like a black rock in space with no atmosphere. Speculate as to why the Earth is warmer than this steady-state calculation indicates: