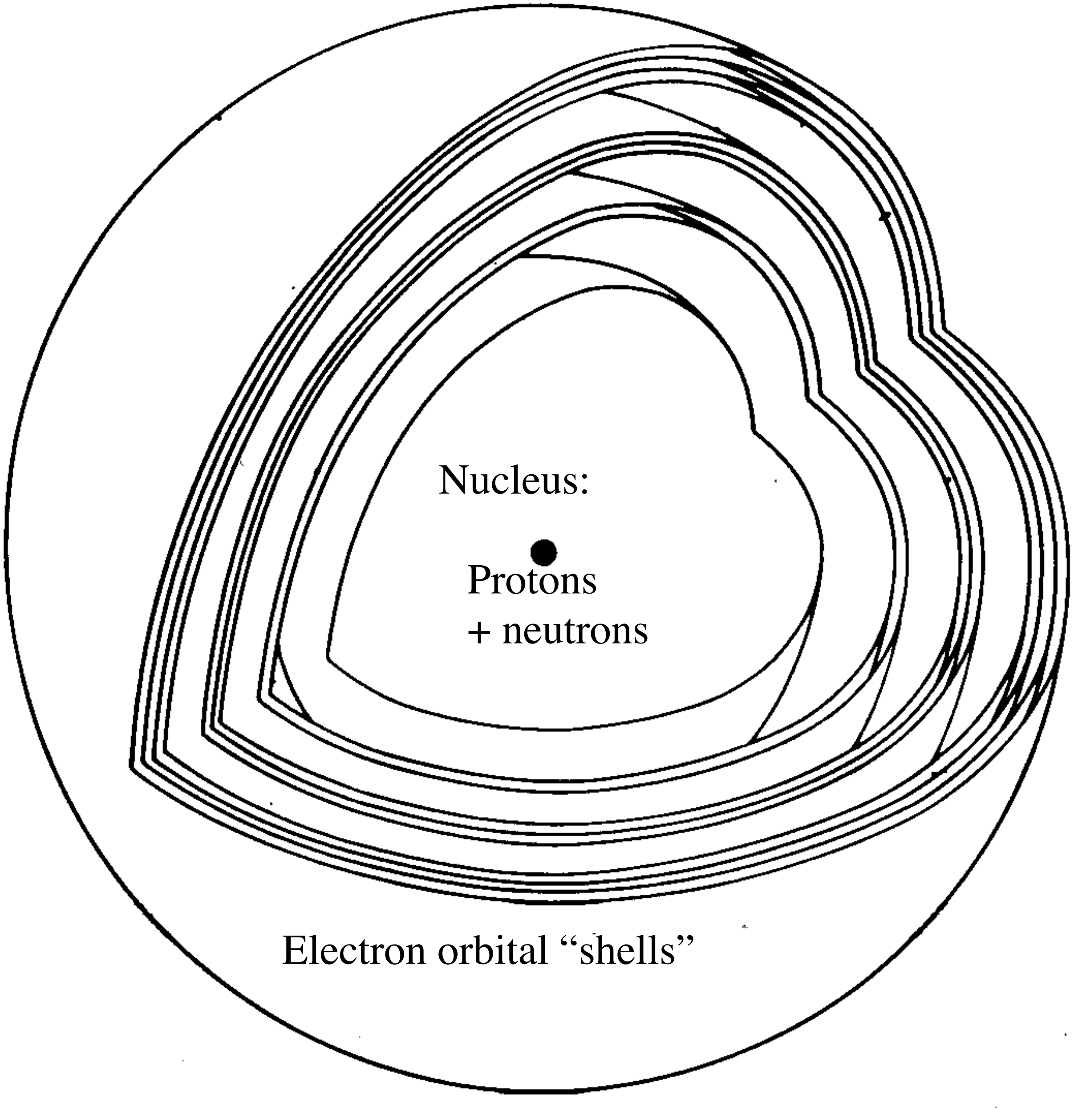
Atoms, nuclear forces, radioactive decay

The basic plan…

- Atomic Structure
- Radioactive Decay
- Fusion in Stars
- Formation of Elements


Nucleus:
Protons
+ neutrons
Electron orbital "shells"

Atoms

- Nucleus very small compare to the size of an atom;
- An ^1H atom diameter = 4×10^{-10} meters
- For ^1H, nucleus diameter is ~1.6×10^{-15} m
- Nucleus contains most of the mass, positive charge
- Electrons have negative charge, little mass
- Electrons and particles governed by quantum mechanics, not Newtonian mechanics.

Atoms

- Neutrons – similar mass as protons
- The number of protons in the nucleus determines the charge → electron orbitals → element with specific chemical behavior
- Chemical reactions are simply those in which electrons on one atom interact with those on other atoms.
- # protons in H? He? Li? C? Fe? U?

Atoms

- The number of protons PLUS number of neutrons determines atomic mass.
- ^{1}H has 1 proton, 0 neutrons (hydrogen)
- ^{2}H has 1 proton, 1 neutron (deuterium)
- ^{3}H has 1 proton, 2 neutrons (tritium)
- ^{4}He has 2 protons, 2 neutrons (helium)
- ^{12}C has 6 protons, 6 neutrons (carbon)
- ^{13}C has 6 protons and …
- ^{14}C is radioactive and has 6 protons and …

Nuclear Forces

- Electrostatic attraction and repulsion is very strong
- You have to accelerate two protons up to the equivalent of ~10,000,000 K temperature to get them to fuse into helium.
- …and yet helium is a very stable nucleus.
- What keep protons together in large atoms?
- Nuclear Strong Force

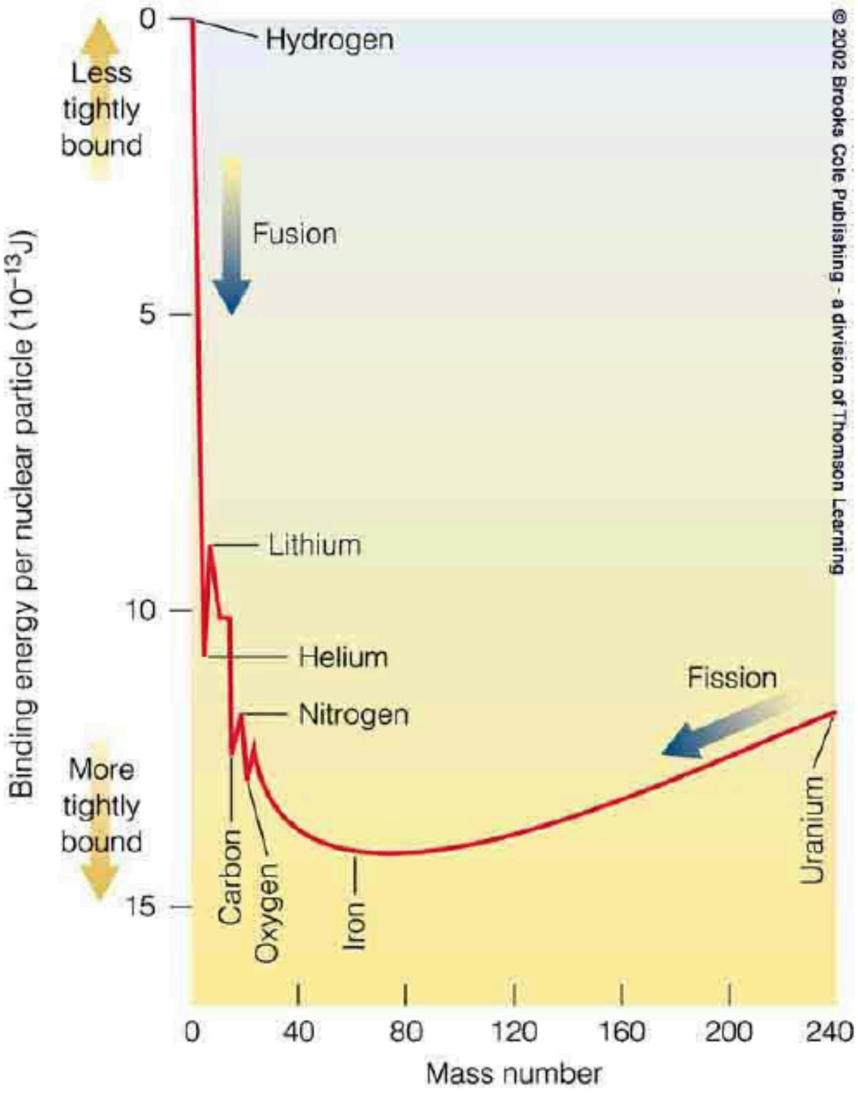
Nuclear Forces

- Nuclear strong force overcomes the repulsion between nuclei, and allows them to stick together.
- Why don’t atoms collapse together?
- NSF is VERY SHORT RANGE – it only affects things within a few proton diameters, so we don’t feel it outside the nucleus.
- Electrostatic repulsion is long range ($1/r^2$)

Nuclear Forces

- At large distances, only Coulomb (electrostatic forces) felt (protons repel)
- At short distances in and very near the nucleus, the NSF is very important “glue”.
- Protons and Neutrons are both “nucleons”, particles that occupy the nucleus and give it mass.

The Curve of Binding Energy



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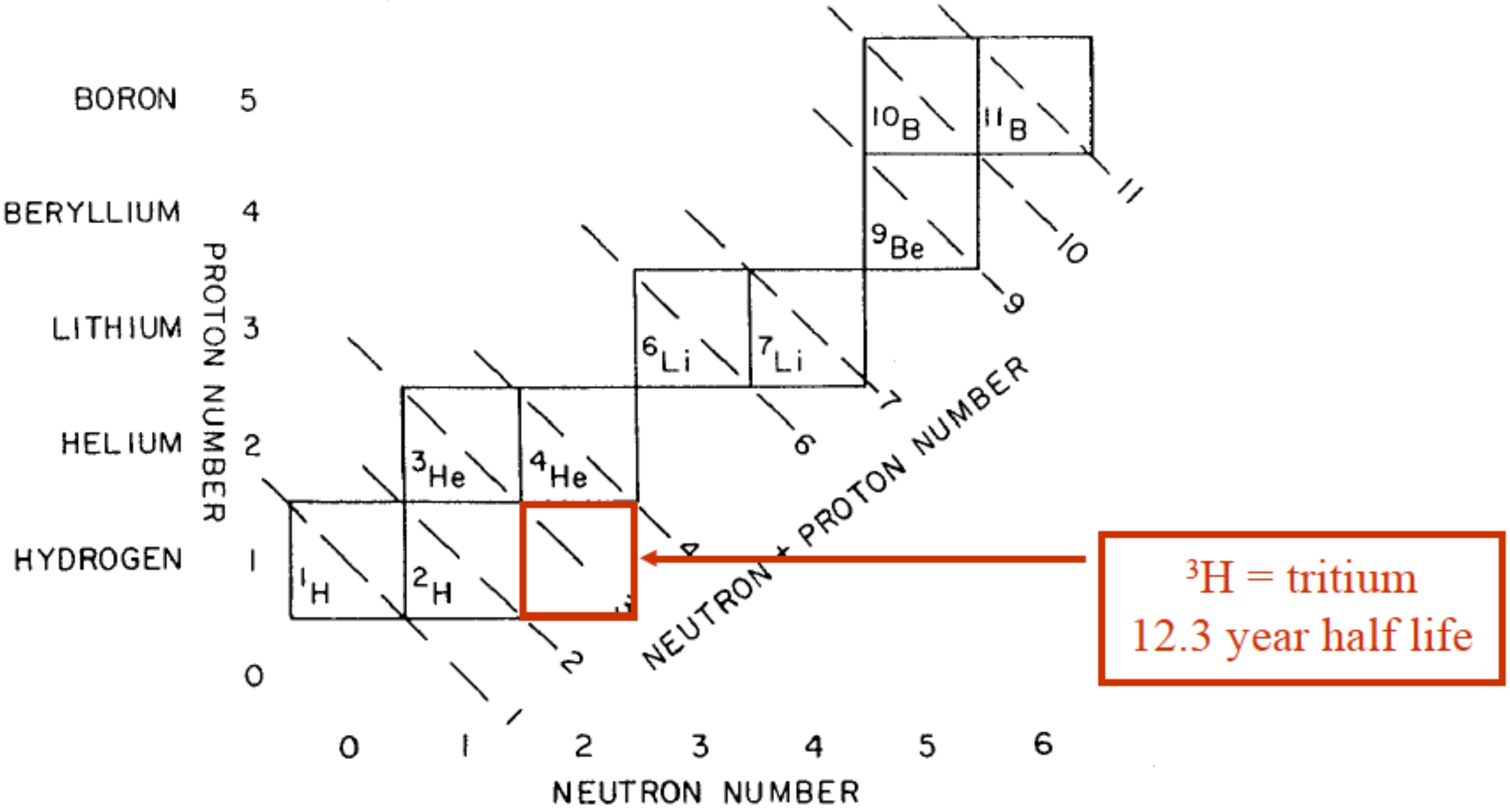
- If you keep adding protons to a nucleus?
  - Coulomb repulsion continues to increase
    - new proton feels repulsion from all other protons
  - Strong force attraction reaches limit
    - new proton can't feel attraction from protons on far side of a big nucleus

- Gain energy only up to point where Coulomb repulsion outweighs strong force attraction.
- Most "stable" nucleus is ^{56}Fe
(26 protons, 30 neutrons, 56 total)

- Release energy by fusion of light nuclei to make heaver ones– up to ^{56}Fe
- Release energy by fission of heavy nuclei to make lighter ones – down to ^{56}Fe

Chart of the nuclides



^{3}H = tritium
12.3 year half life

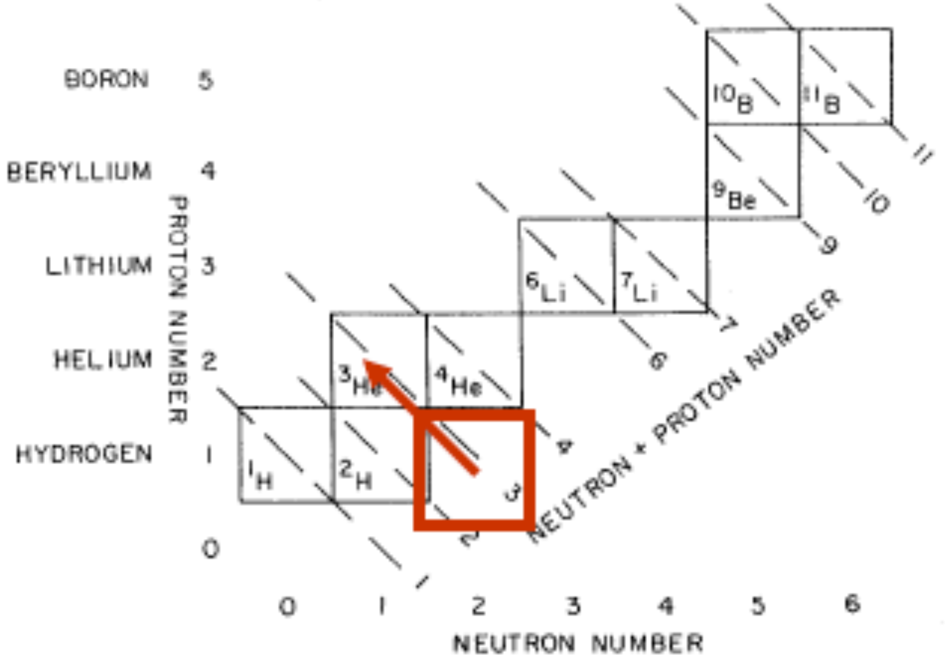
From Broecker (1985) How to Build a Habitable Planet

- Vertical Axis: Number of protons – so different elements
- Horizontal Axis: Number of neutrons – so different isotopes

“Beta” decay

- In the ^{3}H nucleus, one of the neutrons “decays”:
- N → $p^+ + e^- + \nu$
- The proton (p^+) stays in the nucleus, the electron (e^-) is ejected from the nucleus, along with a nearly mass-less neutral particle called a neutrino (ν)
- (When radioactivity was first discovered, three “things” were observed coming from decaying nuclei: “beta” particles, “alpha” particles, and “gamma” rays”. “Beta” particles later turned out to be simply electrons)
- Essentially, a neutron becomes a proton, and we have made a nucleus with 2 protons now, helium…

- The old ^{3}H nucleus has 1 proton, 2 neutrons
- The new nucleus has 2 protons and 1 neutron
- This makes the new one ^{3}He
- The number of protons goes up, the number of neutrons goes down:



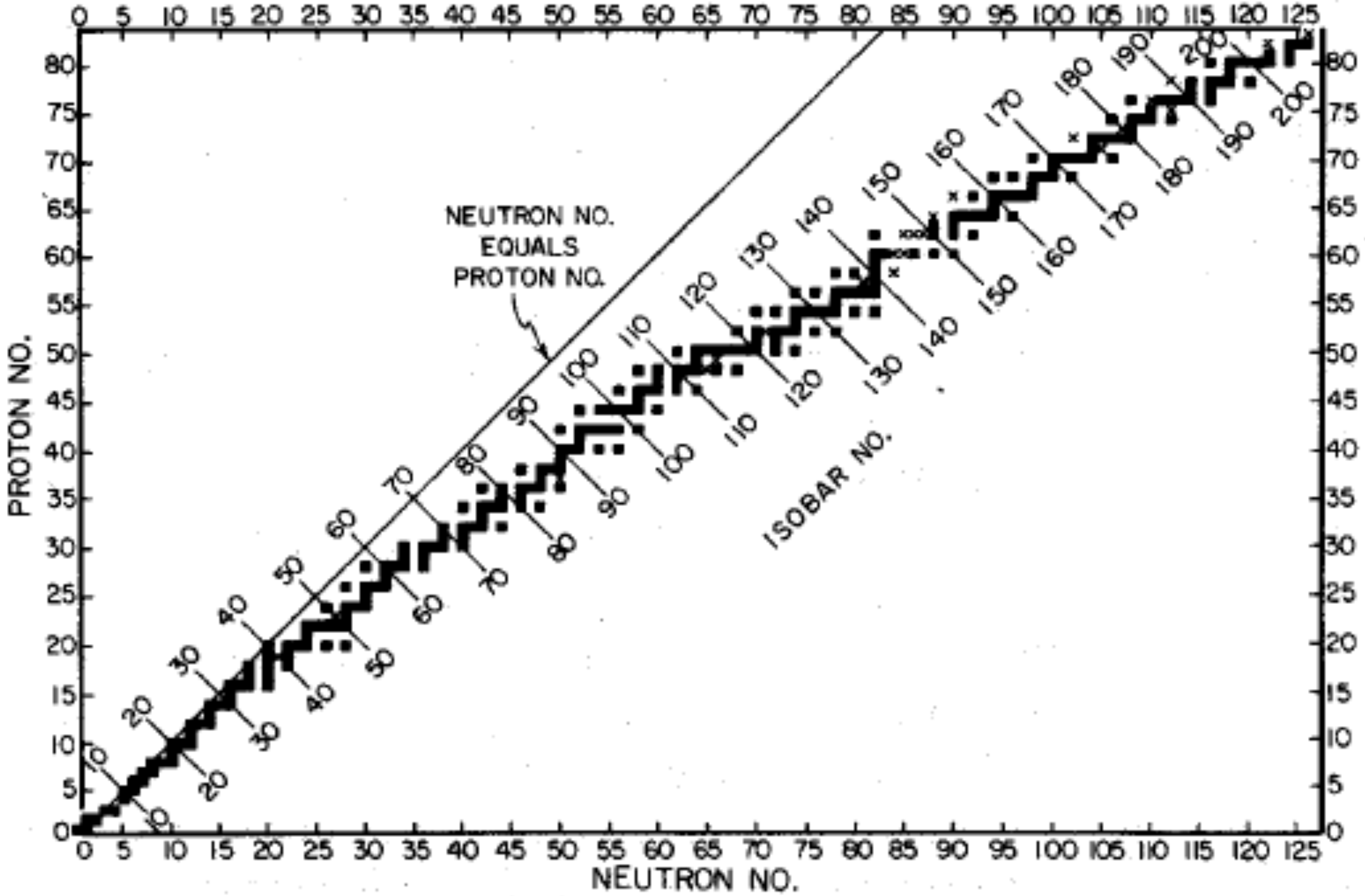
Four Forces:

• Gravity	Astronomical Scale
• Electromagnetic	Holding atoms together
• Nuclear Strong Force	Holds nuclei together
• Nuclear Weak Force	$n \leftarrow\rightarrow p^+, e^-$ radioactive decay

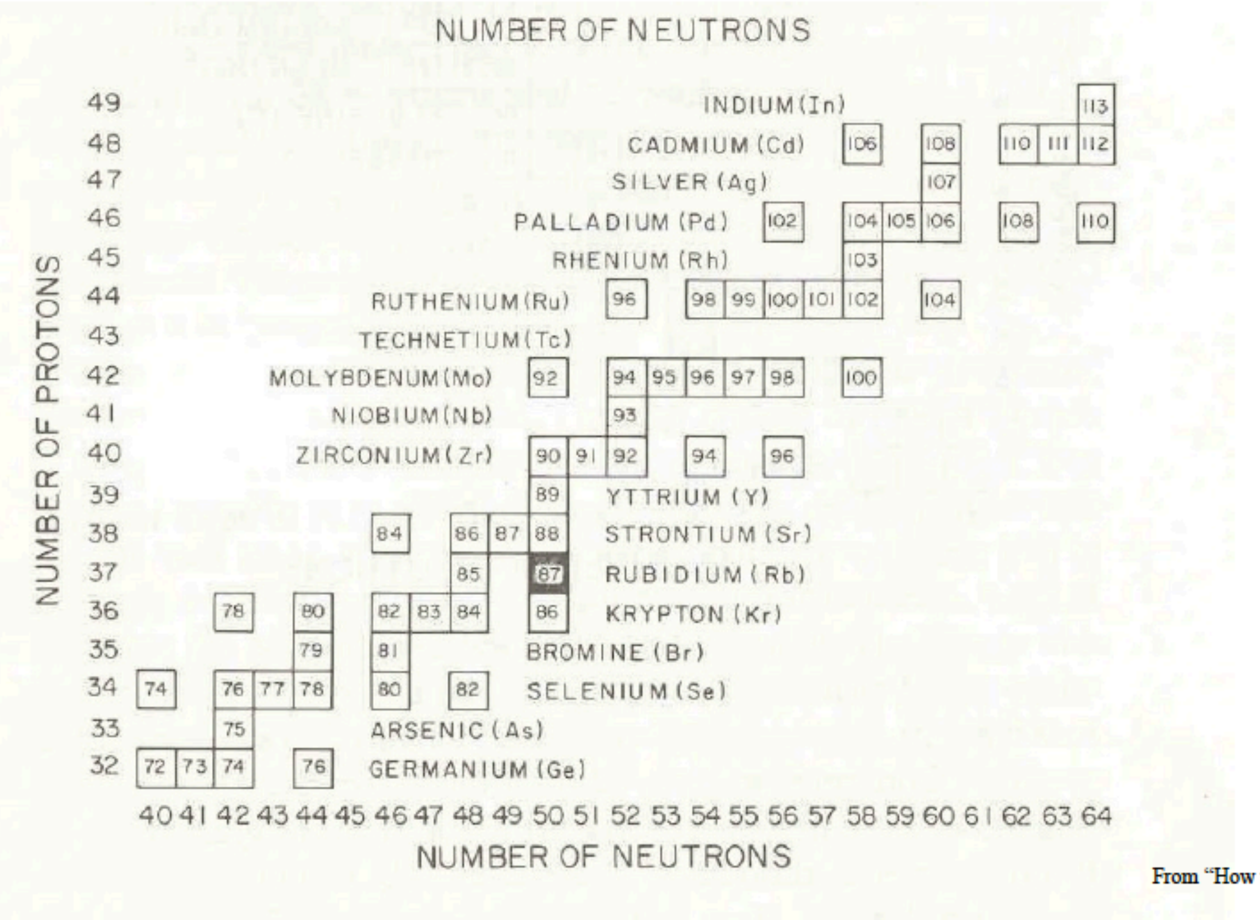
Weak Force…

- Why not just keep adding neutrons to nuclei?
- They have strong-force binding, but no Coulomb (electrostatic) repulsion, should get a lot of energy
- Weak force plays a role in allowing neutrons to beta decay and become protons; weak force tends toward equal number of neutrons and protons, but with so much Coulomb repulsion in large nuclei, more neutrons than protons are needed.

Larger nuclei need more neutron “glue”



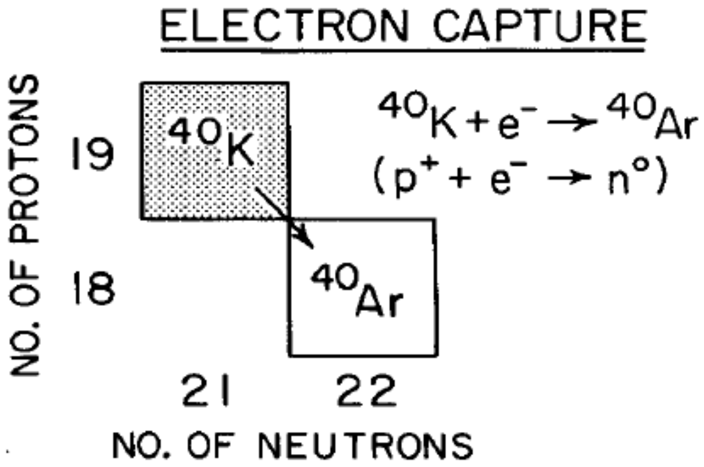
Rubidium to Strontium Beta Decay



From "How to Build a Habitable Planet" by Broecker

- Suppose you have too many *n* compared to p^+
- Over time inside ^{87}Rb one $n \rightarrow p^+$ so $^{87}\text{Rb} \rightarrow {}^{87}\text{Sr}$
- We'll use relative amounts of Rb, Sr to determine age

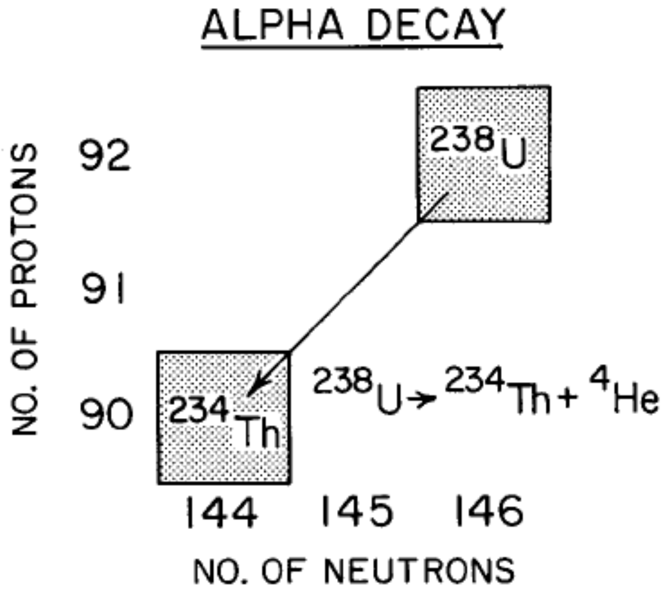
Electron Capture



From “How to Build a Habitable Planet”
by Broecker

- The innermost electron reacts with a proton, turning it into a neutron
- With one more neutron and one less proton,
  you move towards lower right in chart of nuclides
- Once again the “atomic mass” does not change – it’s still “40”

Alpha Decay



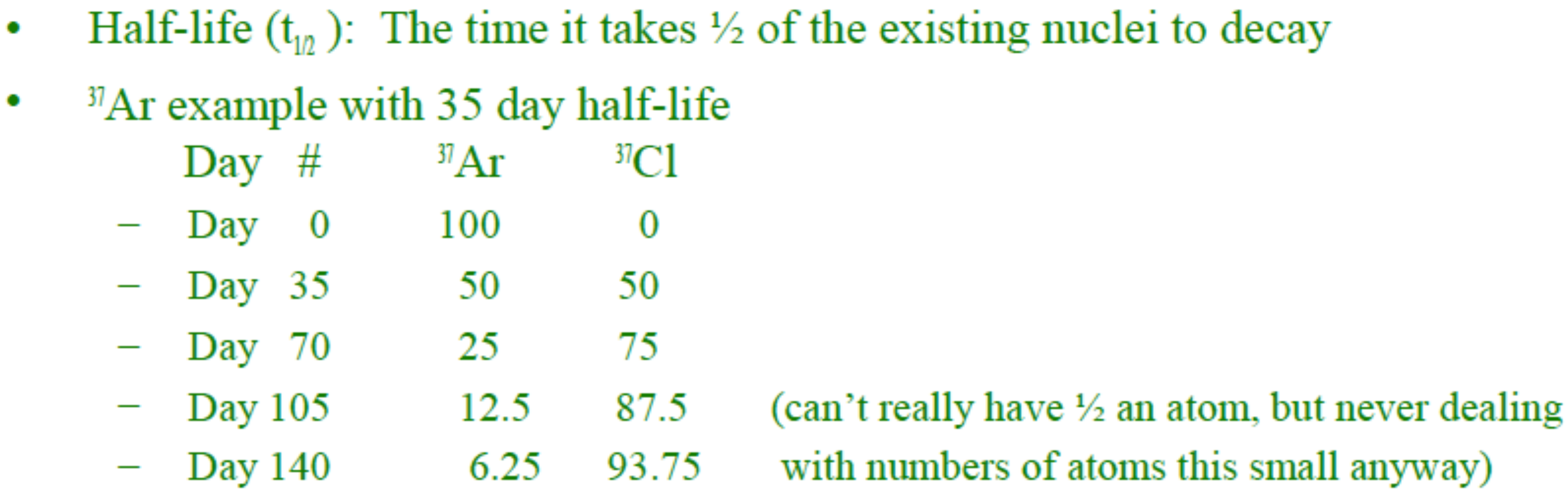
Corrected, (neutron and proton numbers were originally swapped)
from "How to Build a Habitable Planet"by Broecker

- Ejection of an "alpha particle" = (2 neutrons + 2 protons) = ^{4}He nucleus
- Weak force <u>not</u> involved in this type decay since no n⇔$p^+ + e^-$
- Just fight between coulomb repulsion and strong (nuclear) force

Beta decay, Electron capture, Alpha decay

Half Life

- $^{37}Ar \rightarrow {}^{37}Cl$, half-life = 35 days
- Decay rate is independent of chemical and other conditions outside the nucleus.



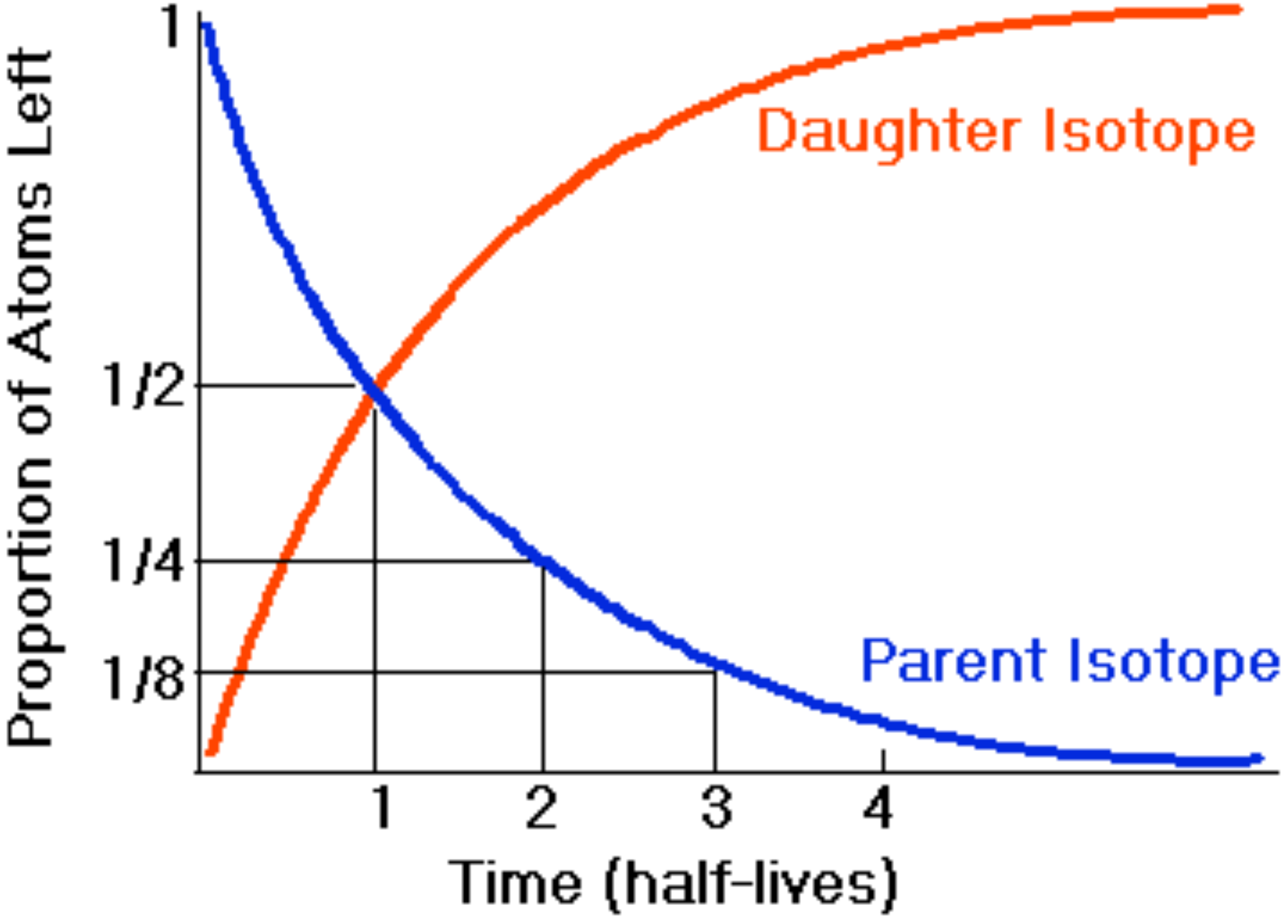
- Half-life ($t_{1/2}$): The time it takes ½ of the existing nuclei to decay
- ^{37}Ar example with 35 day half-life

Day #	^{37}Ar	^{37}Cl	
– Day 0	100	0	
– Day 35	50	50	
– Day 70	25	75	
– Day 105	12.5	87.5	(can't really have ½ an atom, but never dealing
– Day 140	6.25	93.75	with numbers of atoms this small anyway)

- Using power of 2:

$$
\begin{aligned}
N_{\text{parent}} &= N_{0\ \text{parent}} \times \left(\frac{1}{2}\right)^{t/t_{1/2}} \\
&= N_{0\ \text{parent}} \times 2^{-t/t_{1/2}} \\
N_{\text{daughter}} &= N_{0\ \text{daughter}} + \left(N_{0\ \text{parent}} - N_{\text{parent}}\right) \\
&= N_{0\ \text{daughter}} + N_{0\ \text{parent}} \times \left(1 - 2^{-t/t_{1/2}}\right)
\end{aligned}
$$

Graphically:



Decay constants

- Powers of two are less convenient mathematically than exponentials and logarithms. Although geologists use “half-lives” in popular talks, they use “decay constants” in their own work.

$e = 2.718282\ldots\ldots$ is another *transcendental* number, like π

$\exp(x) \equiv e^x$ so $\exp(1) = e^1 = 2.7182$ $\exp(2) = e^2 = 7.3891$ etc.

$\ln(y) \equiv x$ where x is the number such that $e^x = y$

Natural logrithms and exponents are entirely analagous to base 10 ones except they use e instead of 10.

$\log(y) = x$ where x is the number such that $10^x = y$

$10^0 = 1$ $10^1 = 10$ $10^2 = 100$ $\log(10) = 1$ $\log(100) = 2$

$e^0 = 1$ $e^1 = 2.7182$ $e^2 = 7.3891$ $\ln(2.718) = 1$ $\ln(7.3891) = 2$

$2 = e^{\ln(2)}$ and $2^x = \left(e^{\ln(2)}\right)^x = e^{x\ln(2)}$ so

$$2^{-t/t_{1/2}} = e^{-\frac{t}{t_{1/2}}\ln(2)} = e^{-t\frac{\ln(2)}{t_{1/2}}} = e^{-\lambda t} \quad \text{where} \quad \lambda \equiv \frac{\ln(2)}{t_{1/2}} = \frac{0.6931}{t_{1/2}}$$

where λ is called the *decay constant*.

$$N_{\text{parent}} = N_{0\ \text{parent}} \times 2^{-t/t_{1/2}} = N_{0\ \text{parent}} \times e^{-\lambda t}$$

$$N_{\text{daughter}} = N_{0\ \text{daughter}} + N_{0\ \text{parent}} \times \left(1 - e^{-\lambda t}\right)$$

Decay constant examples

- Example using ^{37}Ar after 70 days (2 half-lives)

$$\lambda \equiv \frac{\ln(2)}{t_{1/2}} = \frac{0.6931}{t_{1/2}} = \frac{0.6931}{35\ \text{days}} = 0.0198\ \text{day}^{-1} \quad \text{for } ^{37}\text{Ar}$$

After 70 days

$$N_{\text{parent}} = N_{0\ \text{parent}} \times e^{-\lambda t} = 100 \times e^{-0.0198\ \text{days}^{-1} \times 70\ \text{days}} = 100 \times e^{-1.386} = 100 \times 0.250 = 25$$

$$N_{\text{daughter}} = N_{0\ \text{daughter}} + N_{0\ \text{parent}} \times (1 - e^{-\lambda t}) = 0 + 100 \times (1 - 0.250) = 0 + 100 \times 0.75 = 75$$

- $e^{-\lambda t} \rightarrow 1$ as $t \rightarrow 0$ and $e^{-\lambda t} \rightarrow 0$ as $t \rightarrow \infty$ so the initial numbers work out right and the final numbers become

$$N_{\text{parent}} \rightarrow 0$$

$$N_{\text{daughter}} \rightarrow N_{0\ \text{daughter}} + N_{0\ \text{parent}} \times (1 - 0) = N_{0\ \text{daughter}} + N_{0\ \text{parent}}$$

Ages from simple decay – no initial daughter element present

- If you can be sure that initially none of the daughter is present then getting ages is simple. In most cases it would be incorrect to assume no daughter is present when the rock solidifies. However in some crystals, for example zircon, a parent like uranium is common but the daughter, lead, is excluded.
- Suppose we have bottle of ^{37}Ar which is decaying to ^{37}Cl. We want to know how long ago the ^{37}Ar was isolated and we <u>assume</u> that there was no ^{37}Cl contamination in the bottle when we started.

  Suppose we find that the amount of ^{37}Ar = 30 and the amount of ^{37}Cl = 70.

$$N_{\text{parent}} = N_{0\ \text{parent}} \times e^{-\lambda t}$$

$$N_{\text{daughter}} = N_{0\ \text{daughter}} + N_{0\ \text{parent}} \times (1 - e^{-\lambda t}) = 0 + N_{0\ \text{parent}} \times (1 - e^{-\lambda t})$$

$$\frac{N_{\text{daughter}}}{N_{\text{parent}}} = \frac{N_{0\ \text{parent}} \times (1 - e^{-\lambda t})}{N_{0\ \text{parent}} \times e^{-\lambda t}} = \frac{1 - e^{-\lambda t}}{e^{-\lambda t}} = e^{\lambda t} - 1$$

$$e^{\lambda t} = 1 + \frac{N_{\text{daughter}}}{N_{\text{parent}}}$$

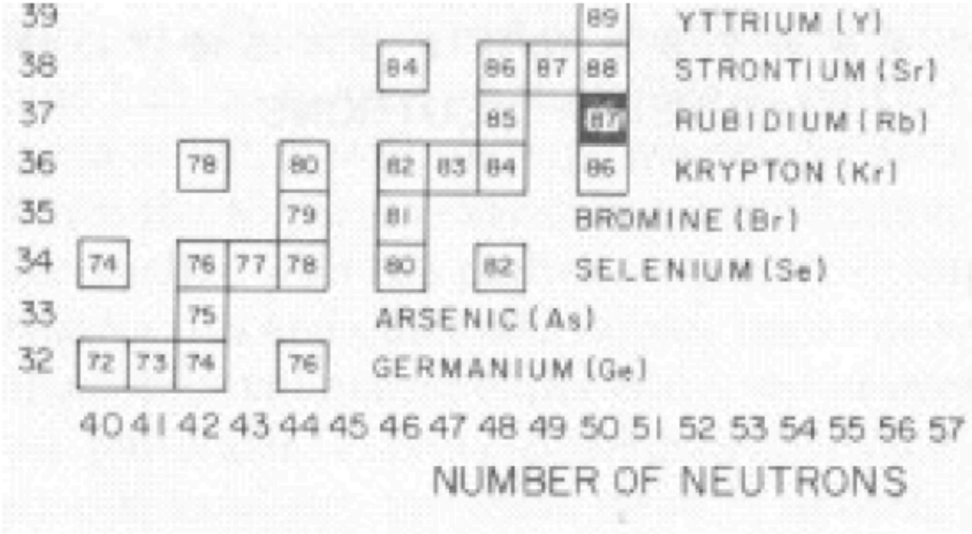
$$\ln(e^{\lambda t}) = \ln\left(1 + \frac{N_{\text{daughter}}}{N_{\text{parent}}}\right)$$

$$\lambda\ t = \ln\left(1 + \frac{N_{\text{daughter}}}{N_{\text{parent}}}\right)$$

$$t = \frac{1}{\lambda} \ln\left(1 + \frac{N_{\text{daughter}}}{N_{\text{parent}}}\right) = \frac{1}{0.0198\ \text{day}^{-1}} \ln\left(1 + \frac{70}{30}\right) = \frac{1}{0.0198\ \text{day}^{-1}} \ln(3.333) = \frac{1}{0.0198\ \text{day}^{-1}} 1.204 = 60.8\ \text{days}$$

Note this is just a little less than two half-lives, when the numbers would reach 25 and 75 .

^{87}Rb → ^{87}Sr ages



From "How to Build a Habitable Planet"
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How do we know how much ^{87}Sr was present when rock formed?

1) Find a mineral which incorporates lots of Rb but virtually no Sr, then assume initial ^{87}Sr = 0. First approximation, but not very accurate.

2) Compare quantitatively results for different minerals which incorporate different relative amounts of Rb and Sr, then learn how to "extrapolate" to ^{87}Sr=0

Sr comes in a radiogenic form ^{87}Sr, plus a non-radiogenic form ^{86}Sr. They behave identical chemically so when the rock formed from the melt, it is homogenized and all minerals will have the same ^{87}Sr/^{86}Sr ratio as the whole rock. Then as time goes by those minerals which have more ^{87}Rb will become progressively more ^{87}Sr rich.