Isostasy and Large Scale Gravity Chap. 9 Homework Answers (Dec. 2009)

1. **Why does it seem a contradiction that the mantle can both transmit shear-waves (S-waves) and rebound isostatically? Explain on both an atomic and a bulk scale.**

   The false paradox here is that how can the mantle both undergo 'solid-state' creep (i.e., flow like a liquid) due the application of loads (e.g., ice sheets, erosion) over millions of years AND transmit earthquake induced shear waves with wave periods of <100 s. The logic goes, if the earth behaves as a 'liquid' to long time scale load, and hence is 'like a liquid' internally, then since (ideal) liquids have no shear strength, then no earthquake shear waves should propagate through the earth. This was a big debate after earthquakes shear waves were finally measured in 1900!

   At the atomic scale (10^{-9} m), earthquake induced shear wave oscillations travel through the earth as 'elastic waves'. For these short wave periods waves (<100 s), this means that the atoms can be well modeled as mass points with springs connecting the atoms as dictated by the crystal lattice arrangement of atoms. This means that shear waves can propagate through the atom 'springs' (atomic bonds) without much lose (dissipation) of the kinetic and strain energy associate that IS the seismic wave. Thus, the waves from an earthquake larger than magnitude 5 will be detectable by a seismometer on the other side of the planet.

   But, at the atomic scale, the addition or subtraction of mass loads (e.g., ice sheets, filling sediment basin) occurs very long time scales (i.e., million years) much longer than the time-scales for earthquakes waves (<100 s wave period). Thus, on the long time scales, the physics shows that vacancy within a solid will slowly drift in the direction of the stresses applied by the load causing rock strain. This permits the deep hot rock to 'flow like a liquid' so that the earth's surface will go up and down by hundreds of meters over millions of years.

2. **How would you tell if an area is in isostatic equilibrium?**

   Isostasy from its Greek root means "same standing" implying gravitational force equilibrium between different height blocks. The force derives from the 'pull' of gravity upon lateral variations in the density (mass) of the lithospheric blocks. Thus, isostatic equilibrium is the same as gravitational equilibrium. When a region is said to be in isostatic equilibrium, we are assuming a model of the earth which says that rigid (infinite strength) blocks of lithosphere are floating in the 'liquid-like' asthenosphere that will flow when surface loads (mass anomalies) are applied over time. Using this 'rigid-block over a liquid-like asthenosphere' as our Isostasy model, this predicts that lateral variations in topography are compensated by either mass deficits below high standing regions (roots) or mass excesses below low standing regions (anti-roots). Put another way, when isostatic equilibrium is operative, then changes in elevation (height) manifest changes, or we say are compensated by, subsurface density variations (roots or anti-roots). Thus, the free air gravity measured over different elevation (height) areas that are in isostatic equilibrium will have the same free-air gravitational values! Amazing.

3. **A large continental area covered with ice has a positive gravity anomaly. Which of the following might account for the positive gravity anomaly?**

   (i) The thickness of ice increased recently.
   Yes. Assuming recently means less than 10,000 yrs, then solid-state creep would not be able to establish isostatic equilibrium from the added ice thickness. In fact, the new ice would make for isostatic **dis-**equilibrium. Because the extra ice would be extra mass that is not isostatically compensated, the new ice would makes extra (higher) gravity.

   (ii) The thickness of ice decreased recently.
   No. This would make a negative gravity anomaly. The argument is the opposite of answer (i). We would be in isostatic dis-equilibrium and the reduced ice thickness (mass) would create less gravity.

   (iii) The thickness of ice increased several tens of thousands of years ago.
   No. The 20,000 yrs since the application of the ice load (mass added) would permit isostatic equilibrium; hence, the free-air gravity anomaly would be zero.

   (iv) The thickness of ice decreased several tens of thousands of years ago.
   No. The 20,000 yrs since the application of the ice (un)load (mass removed) would permit isostatic equilibrium; hence, the free-air gravity anomaly would be zero.

4. **Explain how erosion of mountains can sometimes result in uplift of the peaks.**

   Tricky question. Consider a flat uplifted plateau that is then eroded by rivers due to increased precipitation. As the rivers down cut the valleys, the river water will carry away the sediment, hence mass, of the plateau. This removal of mass unloads (removes mass) from the lithospheric block. This change in mass permits the plateau to isostatically rebound: just like the shorelines that move up when an ice sheet melts. Finally, if the mountain tops erode more slowly than the river valleys, then the net effect from the isostatic rebound driven by the eroding valleys, will be for the mountains to get temporarily higher (uplifted).

5. **Melting of the ice in the Arctic (not Antarctic) region, where the ice floats on the sea, would cause the sea level to (i) rise (ii) fall (iii) be unchanged.**

Deep thought question. Given that the Arctic ice is a rigid block (the ice) which is floating in a liquid (the ocean), we can assume isostatic equilibrium is operative. Isostatic equilibrium implies that the mass of the ice above the sea level is proportional to the density difference between the ice and the liquid. Note that the density contrast between the ice and water is key and because the ice is less dense than water, the ice floats with a small portion (about 10%) of the icebergs total height being above the water-line. Thus, the mass of ice above water is compensated by the lower density (0.9 Mg/m^3) of the iceberg's root below the waterline with respect to water (1.0 Mg/m^3).

Thus, when the solid form of water (ice) melts via a phase change to make the liquid form of water, its density changes (increases). The density of liquid water is greater than solid ice because the liquid water molecules are closer together than ice molecules. This means that a mass of ice, when melted occupies a smaller volume in its liquid form! Of course, the mass does not change because mass in conserved for all non-nuclear reactions. Thus, the 'extra' volume of ice associate with the floating arctic ice sheet, when changed to its liquid form (melts), creates no volume change of the ocean: hence, no sea-level change!

Go do an experiment at home: put an ice cube in a cup, mark the water level, let the ice melt, and see if the water level changed.

6. **If all the ice of Antarctica were to melt rapidly, would you expect (a) a thousand years later, (b) a million years later, that the shoreline around Antarctica would be, compared to the present, higher, lower, or the same ?**

   Note that Antarctica has about 80% of the worlds fresh water ice that is grounded (not floating). So, if all of it melted, the global sea level would rise about 70 m.

   (a) Higher. The sea level would rise 70 m and isostatic rebound would not occur on a 1000 year time-scale.
   (b) Lower. The isostatic rebound (uplift) from the melting of the ice-sheet 1 Ma ago would be greater (200 m) than the 70 m increase in sea-level. Isostatic rebound wins!

7. **If a continental area is in perfect isostatic equilibrium, which of the following is true?**

   (i) A Bouguer anomaly map would show no variations.
   False. A region can be in perfect isostatic equilibrium and still show variations in the Bouguer gravity map due to the mountain roots and depression anti-roots. The free air gravity map would show no variations though!

   (ii) There are no lateral variations of density below the surface.
   False. Isostatic equilibrium requires lateral variations in the subsurface density to compenenstate the variations in elevation.

(iii) **No uplift or subsidence is occurring.**
True. If uplift or subsidence is actively occurring at a significant rate (1 mm/yr) then isostatic compensation, due to mantle flow, will not have time to occur. Hence the mass distribution will not be compensated properly. Actively shortening mountain ranges can create mass excesses that are uncompensated and hence make more gravity than if they were compensated.

8. **A wide block of wood 100 cm high and with density of 0.72 Mg/m^3 is floating in a liquid with density of 0.96 Mg/m^3.**

(a) Calculate how far the top of the blocks is above the liquid surface.

The liquid density is ρ_l = .96 Mg/m^3 and the wood density is ρ_w = .72 Mg/m^3. Call the height above the block above the liquid surface h_1 and the thickness of the part of the block below the surface h_2. Thus, we have the height equation

$$h_1 + h_2 = 100\ cm.$$

Now construct the weight (force) balance equation:

$$g*(h_1\rho_w + h_2\rho_w) = g*(h_2\rho_l) \text{ (N/m}^2\text{) or Force per unit area.}$$

This equation can be simplified to:

$$\rho_w(h_1 + h_2) = \rho_l h_2$$

We know have two equations and two unknows; this system can be algebraically solved for both h_1 and h_2 . To do this, substitute in for one variable in terms of the other variable to isolate one equation with only one unknown. For example, rearrange the height equation and substitute that into the weight equation:

$$h_2 = 100 - h_1$$

$$\rho_w(h_1 + (100 - h_1)) = \rho_l h_2$$

$$\rho_w 100 = \rho_l h_2 \quad \gg \quad h_2 = 100*\frac{\rho_w}{\rho_l} = 100*\frac{0.72}{0.96} = 75\,cm$$

So, substituting into the height equation arranged to isolate h_1 gives:

$h_1 = 100 - h_2 = 100 - 75 = 25\ cm$ (height of the wood block above the surface)

(b) How far would it be if 12 cm were removed from the base of the block.

$$h_2 = 88 - h_1 \qquad h_2 = 88 * \frac{\rho_w}{\rho_l} = 88 * \frac{0.72}{0.96} = 66\,cm$$

$$h_1 = 88 - h_2 = 88 - 66 = 22 \ cm$$ (height of the wood block above the surface)

(c) How far would it be if 12 cm were from the top of the block.

Same answer as (b). What is relevant is the total volume (hence mass, hence weight) of the block; the volume (mass) of the block is proportional to the height of the block. The height (hence mass) of the block is the same if the block's height is shortened 12 cm on its top or base!

9. A large area of continent consists of 30 km of crust with density 2.8 Mg/m^3 over 90 km of material with density 3.1 Mg/m^3. The asthenosphere density is 3.2 Mg/m^3. This region is covered with a 1.6 km thickness of ice of density 0.9 Mg/m^3. The ice covered region is assumed to be isostatic equilibrium. Then, the ice melts. By how much will the rock surface of the continent change when the new isostatic equilibrium is re-established ?

Define layer densities: ρ_i=0.9 ρ_c=2.8 ρ_m=3.1 ρ_a=3.2 (units= Mg/m^3)
Define layer thickness: t_i=1.6 t_c=30 t_m=90 infinite (units= km)
Define height variables: h_1 is bottom layer offset, h_2 is top layer offset

The **height equation** is: $(90 + 30 + 1.6) = h_1 + h_2 + (30+90)$

Which simplifies to: $1.6 = h_1 + h_2$ (km)

The **weight equation** is: $t_i \rho_i + t_c \rho_c + t_m \rho_m = 0*h_2 + t_c \rho_c + t_m \rho_m + h_1 \rho_a$

Which simplifies to: $t_i \rho_i = h_1 \rho_a$ (note that the common layers between the two blocks always cancel out!)

Solving for h_1 gives: $h_1 = t_i * (\rho_i / \rho_a) = 1.6 * (0.9/3.2) = .45$ km

So h_2 equals: $h_2 = 1.6 - h_1 = 1.15$ km

So, the answer is that the former rock-ice surface that is now the rock-air surface rose 1.15 km upwards towards the stars due to isostatic equilibrium being re-established after the ice sheet melted.

10. The crust of a continent contains a 3 km thick layer of salt (2.2 Mg/m^3) that is embedded with sediments (2.4 Mg/m^3). Over time, the salt layer flows laterally and

the layer thins to 1 km thickness. Assume an asthenosphere density of 3.2 Mg/m³ . The 2 km thinning of the salt layer cause the continental surface to lower by how much?

Define layer densities: salt = $\rho_s = 2.2$ asthenos = $\rho_a = 3.2$ (units= Mg/m^3)
Define height variables: h_1 is bottom layer offset, h_2 is top layer offset

Height equation is : 3 (original salt) = h_1 + 1 (final salt) + h_2 (km) (common layers cancel)

Which simplifies to: $h_1 + h_2 = 2$ (km)

Weight equation is : $3*\rho_s = h_2*0.0 + 1*\rho_s + h_1*\rho_a$ (kg/m^2) (common layers cancel)

Which simplifies to: $h_1 = 2*(\rho_s / \rho_a) = 1.38$ km

Solving for h_2, the surface subsidence after 2 km of salt was squeezed out, gives :

$$h_2 = 2 - h_1 = 0.62 \text{ km or } 620 \text{ m.}$$

11. **A large area is intruded by three basaltic sills with thickness of 30, 40, and 50 m. The sill density is 2.8 Mg/m³ and the asthenosphere density is 3.2 Mg/m³ . What is the change in the height of the surface after isostatic equilibrium has been restored?**

Define layer densities: Sills = $\rho_s = 2.8$ asthenosphere = $\rho_a = 3.2$ (units= Mg/m^3)
Define height variables: h_1 is bottom layer offset and h_2 is top layer offset

The total thickness of the sills is 120 m.

Height equation is: $h_1 = h_2$. This simplicity is because no layer thicknesses are changed.

Weight equation is: $120 * \rho_s = h_1 * \rho_a$ (kg/m^2)

Therefore, $h_1 = 120 * (\rho_s / \rho_a) = 105$ m

So, the addition of 120 m of sills to crust, mean the top (and bottom) surface rises by 105 m.

12. **A 2 km deep sea is filled to sea level by sediments over a long period of time. Assuming isostatic equilibrium is maintain, how deep will the sediments be? Use these densities in Mg/m³ units: water (1.00), sediment (2.40), asthenosphere (3.2).**

Define layer densities: $\rho_w = 1$ $\rho_s = 2.4$ $\rho_a = 3.2$ Mg/m^3
Define layer thickness: $t_w = 2$ $t_s = 2 + h_1$ km

Weight equation: $t_w \rho_w + h_1 \rho_a = (2 + h_1) \rho_s$

Solving for h₁: $h_1 = \dfrac{2\rho_s - t_w\rho_w}{\rho_a - \rho_s} = \dfrac{2*2.4 - 2*1}{3.2 - 2.4} = 3.5\ km$.

In additional to filling up the sea with 2 km of sediment, the sinking of the lithosphere due to replacing the water with heavier sediment causes a 3.5 km of extra subsidence and hence 3.5 km of 'extra' sediment. Thus, the original 2 km sea is now a 5.5 km thick sediment basin.

13. SKIP.

14. SKIP.

15. The uplift of former beaches around the Gulf of Bothnia is about 275 m. What thickness of ice would be needed to depress the beaches back to sea level? For densities use: ice (0.9), asthenosphere (3.2) Mg/m³ .

Define: $\rho_i = 0.9\ and\ \rho_a = 3.2\,Mg/m^3$

There are only two density changes: addition of ice (replacing air) to top of block (B) and pushing the base of the block (B) lithosphere into the asthenosphere. The problem states that the former -beach surface should be pushed down 275 m.

Weight equation: $h_1\rho_i = 275*\rho_a \quad \gg \quad h_1 = 275*\left(\dfrac{\rho_a}{\rho_i}\right) = 977\ m.$

16. The value of little-g at a place A is less than that at place B. Which of the following might be the explanation?

(i) A is at a higher latitude.
FALSE: assuming no other gravitational effects are present, from the IGF formula, we know that little-g at higher latitudes increases .

(ii) A is at a higher elevation.
TRUE: little- g is proportional to 1/r^2 which means that gravity decreases by 0.386 mgals per meter moved upwards from the Earth's surface

(iii) A is underlain by low density rocks.
TRUE: g is proportional to the mass column under a gravitational measurement point.

(iv) A but not B was covered by a thick ice sheet millions of years ago.
FALSE: One expects that FULL isostatic equilibrium would be re-established after 'millions of years' and hence would NOT affect the gravity values between sites A and B.

17. The value of little-g at a place varies with time due to which of the following?

(i) Isostatic rebound.

(ii) TRUE. If a region is undergoing isostatic rebound to re-establish isostatic equilibrium, then the gravity field will be changing in time as the distribution of mass change with rebound.

(iii)

(iv) The topography of the continents.

(v) FALSE. While topography (elevation) will affect gravity via the Free-Air correction, it will not vary the gravity over time.

(vi)

(vii) Lateral differences in the composition of rocks.

(viii) FALSE. While lateral diference in rock composition, hence density, will change the spatial variation in gravity, it will not change gravity in time.

(ix)

(x) The earth’s rotation.

(xi) FALSE. If one assumes that the Earth's rotational angular velocity is constant, then gravity effect due to non-inertial reference frame force (Centrifugal force) will not change in time. Albeit, the Earth's rotational angular velocity is acutally slowly decreasing in time as angular momentum is exchanged between the Earth-Moon orbital system (the Moon is moving about a few cm per year away from the Earth).