

Gravity Chapter 8 Homework answers (Dec. 2009)

1. **Given the value of little-g at the equator is 9.78031 m/s², what is the value of gravity at the North Pole or South Pole?**

Gravity varies with latitude according to this International Gravity Formula (IGF) as

$$g(\theta) = \gamma_1(1 + \gamma_2 \sin^2 \theta + \gamma_3 \sin^2 2\theta), \quad \gamma_1 = 9.78031, \quad \gamma_2 = 5.3024e^{-3}, \quad \gamma_3 = 5.900e^{-6}.$$

The latitude at the North Pole is $\theta = +90^\circ$ and the latitude at the South Pole is $\theta = -90^\circ$.

Therefore, the predicted gravity at the North Pole is:

$$g(+90^\circ) = \gamma_1(1 + \gamma_2 \sin^2(90^\circ) + \gamma_3 \sin^2(2 * 90^\circ)) = \gamma_1(1 + \gamma_2 1^2 + \gamma_3 0^2) = \gamma_1(1 + \gamma_2)$$

$$g(+90^\circ) = 9.78031(1 + 5.3024e^{-3}) = 9.8322 \frac{m}{s^2}$$

$$g(+90^\circ) = 9.8322 \frac{m}{s^2} * \frac{1 \text{ mGal}}{10^{-5} m / s^2} = 9.8322e^5 \text{ mgals} = 983,220 \text{ mgals}$$

eQ: State why the gravity may or may not be different at the north and South Pole according to the IGF?

eQ: State the number of significant figure for each of the five numbers in the IGF?

eQ: What MKS units does the terms $\gamma_1, \gamma_2, \gamma_3$ have?

eQ: What units does θ have and what is the difference between radian and degree units?

eQ: What two gravitational effects does the IGF account for?

2. **A horizontal sill that extends well outside the survey area has a thickness of 30 m and density of 0.5 Mg/m³ in excess of the rocks it intrudes. Estimate the maximum depth at which it would be detectable using a gravimeter that can measure to 0.1 mGal.**

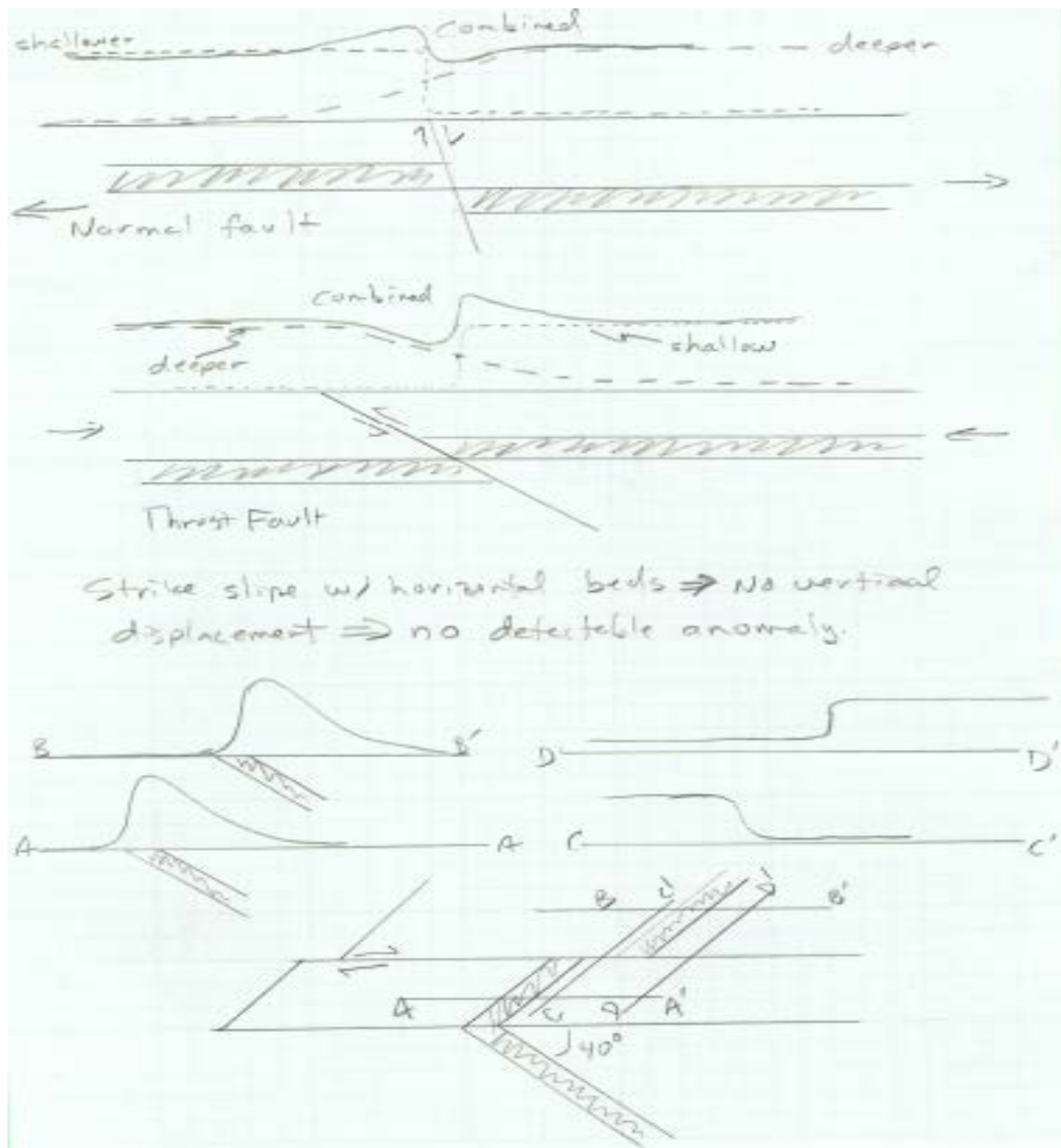
The equation for a flat infinite sheet is: $\Delta g(\Delta\rho, t) = 2\pi G \Delta\rho t$ mGals where G is gravitational constant, $\Delta\rho$ is density contrast, and t is the thickness of the infinite layer. First, we note that the infinite sheet equation does not vary with the depth of the sheet. Hence, the answer is that the depth of the infinite sheet is irrelevant to whether we can detect the gravitational anomaly with a gravimeter accurate to 0.1 mGal.

Second, let us calculate the gravitational pull of the infinite sheet mass anomaly.

$$\Delta g(\Delta\rho, t) = 2\pi G \Delta\rho t = 2\pi(6.672 \times 10^{-8} \frac{m^3}{Mg s^2})(0.5000 \frac{Mg}{m^3})(30.00 m) = 6.288e^{-6} m / s^2 = 0.6288 \text{ mGals}$$

Therefore, our gravimeter with a 0.1 mGal accuracy could detect this anomaly. But, the important point is that the gravity field for an infinite sheet does not change anywhere, it just makes the absolute level of the gravity field 0.63 mGal greater EVERYWHERE! Therefore, the gravity anomaly from the infinite sheet cannot be detected.

3. An extensive dolerite sill was intruded at the interface between horizontal sandstones. Sketch the gravity profiles expected if the sill and beds have been displaced by: (a) A steeply dipping normal fault. (b) A shallow thrust fault. (c) A strike-slip fault. (d) Repeat (c) when the beds dip at about 40'.



4. Calculate how much gravity changes, and whether it is an increase or decrease, on going one km north from the following starting latitudes: (a) equator. (b) 45° N. (c) 45° S. What elevation changes in air would give the same change in g ?

The simplified latitudinal gravity equation for small (<20 km) poleward movements is approximated as:

$$\Delta g_{lat}(\lambda) = 0.812 \sin(2\lambda) \text{ mGal / km} - \text{polewards}$$

An important point to understand is that polewards in northern hemisphere is movement toward the North Pole and polewards in southern hemisphere is movement towards the South Pole. If you are in either hemisphere and move towards the equator that is anti-poleward motion.

a) Equator is $\theta=0^\circ$. Therefore, moving 1 km towards North pole is

$$\Delta g_{lat}(\lambda = 0^\circ) = 0.812 \sin(2 * 0^\circ) \frac{mGal}{km - poleward} * 1 km - north = 0 mGal$$

b) $\theta = 45^\circ$ in North hemisphere. Therefore, moving 1 km toward North pole is motion toward pole

$$\Delta g_{lat}(\lambda = 45^\circ) = 0.812 \sin(2 * 45^\circ) \frac{mGal}{km - poleward} * 1 km - north = +0.812 mGal$$

c) $\theta = 45^\circ$ in South hemisphere. Therefore, moving 1 km towards North pole is motion toward equator

$$\Delta g_{lat}(\lambda = 45^\circ) = 0.812 \sin(2 * 45^\circ) \frac{mGal}{km - poleward} * 1 km - north = +0.812 mGal$$

Now for the tricky part, gravity is a minimum at the equator due to maximal outward directed centrifugal force at the equator due to daily rotation of planet AND the fact that this centrifugal force makes the shape (called figure) of the Earth a flattened ellipsoid. Note that the equatorial radius is about 6,378 km and the polar radius is about 6,356 km. So, to correct for these two factors that change gravity, one must keep track of which way one is moving (i.e., towards pole or towards equator). The gravitational correction is **subtracted** if motion is **poleward** and the gravitational correct is **added** if motion is towards the **equator**. Therefore, (b) answer is -0.812 mGal and (c) answer is +0.812 mGal. This logic is important.

eQ: Why is the centrifugal force directed outwards and why does this decrease gravity ?

eQ: Why is the centrifugal force called a non-inertial reference frame force (also called fictitious force) ?

eQ: Why is the earth not a sphere, but a flattened ellipsoid with an equatorial bulge ?

5. **Why is it more correct to talk of ‘determining the mass of the Earth’ rather than ‘weighing the Earth? Does (a) a spring balance, and (b) a pair of scales measure mass or weight ? State what the balance and scale will measure at the Earth’s surface and in inter-galactic space where we will assume gravity is zero.**

A spring-balance used in the Earth’s gravitational field (e.g., on the surface) measures the force of gravitational attraction between the Planet’s mass and the object’s mass being weighed. The weight of an object is thus ONLY defined in a gravitational field (e.g., the earth and/or near any large massive object such as the Sun and Planets). Where the gravity is near zero (i.e., inter-galactic space), an object’s weight is zero! Therefore, an objects weight changes depending on *where the object is weighed*. A spring-balance works by using Hooke’s law of elasticity which states that the force on a spring is simply equal to the displacement of the spring from its ‘rest’ state times the ‘stiffness’ of the spring.

The gravitational force between two objects, where M_e is the Earth’s mass and M_o is the object’s mass is defined by Newton’s gravitational law

$$\vec{F}(r) = G \frac{M_e M_o}{r^2} \hat{r} \quad \text{Force in Newtons } \left(\frac{kg - m}{s^2} \right).$$

We can define a sphere’s gravitational acceleration field outside its surface simply by leaving out the mass of the object to be weighed.

$$\vec{g}(r) = G \frac{M_e}{r^2} \hat{r} \quad \text{acceleration} \left(\frac{m}{s^2} \right)$$

Evaluation this equation at the Earth's mean radius ($r=6371$ km) gives:

$$\vec{g}(r) = G \frac{M_e}{r^2} \hat{r} \left(\frac{m}{s^2} \right) = (6.67e^{-11} \frac{m^3}{kg \ s^2}) * (5.97e^{24} \ kg) * \left(\frac{1}{(6371e^3 \ m)^2} \right) = 9.81 \frac{m}{s^2}$$

So, if you drop a ball, and ignore air friction, the velocity of the ball increases by 9.8 m/s every second.

Of course, if you know the gravitational acceleration, you can calculate your mass, but ONLY to the accuracy that you know the gravitational field of the planet (which varies by hundreds of mGal).

However, a mass-balance can overcome the 'weighing problem' by NOT measuring the weight of an object, but directly measuring its mass. This is done by 'balancing' an object's mass against known masses placed upon the opposite balance-pan. A mass-balance can perform this miraculous feat of measuring mass, not weight, because the gravitational acceleration that pulls on the object's mass in one pan and the known masses placed in the other pan CANCEL OUT! Note: we are assuming that the gravity field does NOT vary between the two pans of the mass-balance.

So weight is just the gravitational force that any mass 'feels' and your weight depends on where you are with respect to other large masses. More quantitatively, the weight of an object depends on the inverse squared distance between two objects center of masses.

So, now that I know what weight is, then *what is mass?*

Mass is a measure of an objects inertia. And, inertia is a property of all mass (matter) which makes a mass either remain at rest (zero velocity) or remain at a constant velocity UNLESS acted upon by an unbalanced net force. The fact that mass has inertia was only understood in 1594 by Galileo and before that all the physicists (called natural philosophers then) had it wrong! They thought that a ball thrown in the air had to have a force that continued to push on it to keep it moving: e.g., Aristotle said the air around the ball was moving with the ball and that this force applied to the ball, was what keep the ball moving...WRONG! As a corollary to this wrong-headed notion of motion, Aristotle also predicted that the more massive a ball, the quicker it would fall. This is what Galileo finally tested with his experiment that dropped balls from the Tower of Pisa and proved Aristotle to be wrong.

If you really want to know what mass is, the particle physics model QCD-lite predicts that 99% of mass is the three quarks that fly-around each other to make the protons and neutrons that make the atomic nucleus. Since relativity requires that $E=mc^2$, mass is just spatially localized (condensed) energy. Hence, most of the mass of the quarks that make the protons and neutrons is actually angular momentum energy that keeps the quarks ALWAYS bound together and swirling around each other at a scale of 10^{-15} m.

Another definition of mass is simply the number of atoms present in a given volume. That is called the molar mass. We can calculate this approximately because the atomic masses are well-know: just add up the mass of the protons and neutrons that make the nucleus (the electrons weigh nothing compared to the nucleons).

But, most importantly, remember that mass is a measure of inertia; and, inertia means that you need to apply an unbalanced net force to an object to change its state of motion, whether that motion be at rest or moving along a straight line at constant velocity.

So, what then is a force?

A force is ALWAYS and interaction between two (or more) objects that changes their motions. Force is not conserved like energy or momentum. During the interaction (think of two billiard balls colliding), force is ALWAYS an action-reaction pair. That is, the force of object 1 ON object two is equal and opposite (in a vector sense) to the force of object 2 ON object 1. This MUST be true, otherwise the momentum (mass * velocity) of the two balls is NOT conserved (i.e., conserved means the momentum remains constant). This is Newton's third law of motion published in 1687.

Physicist's now know that there are only **four** fundamental forces: gravity, electro-magnetic, and the two forces that are only significant in the nucleus of atoms (the strong and weak forces). From the gravity and electro-magnetic fundamental forces we can derive the concept of pressure and stress in a solid/liquid/gas which are called '**contact forces**'. For example, when two balls collide, the fundamental force at work is the electro-magnetic repulsion between the electron clouds that surround the atomic nuclei at the two balls surfaces. BUT, we can ignore this fundamental force view and just called the force between the electrons a contact force which is the pressure/stress that is impressed during the two balls collision. **Force at distance** also occurs for gravity and electro-magnetism. In essence, a force-field does extend through both matter (the earth) and empty space allowing objects to exchange force without ever touching each other! This concept of forces without contact gave the physicist's real fits for a long time and was not accepted until the 1890's.

eQ: if an object with mass $m_1=1$ kg with a finite speed, collides with a very heavy mass $m_2=1e^{12}$ kg at rest, what happens ? (Use Newton's second law).

eQ: So, if my body mass has the force of the earth's gravity pulling me down, why I am not accelerating ? You are not accelerating downwards because the floor is pushing back against gravity's force to make the net Force zero. Of course, when you jump in the air, and find that it is the gravitational field of the earth that decelerates your motion on the way up and accelerates your motion on the way down.

eQ: When I drop a ball at the Earth surface, the ball clearly falls (accelerates) towards the earth's center of mass, but the Earth does not 'seem' to move towards (accelerate) the ball. Newton's gravitational equation says the Force of the earth on the ball is equal and opposite to the force of the ball on the earth.

eQ: Prove mathematically using Newton's gravity law (Eq. 8.2) and Newton's 2'nd law ($F = m*a$) that two balls of different mass fall at that same rate (ignore air friction).

eQ: Assume you are in a place where gravity is zero, how could you measure an objects mass ? (Hint: use Newton's second law).

eQ: Is the inertial mass used in Newton's second law the same as the gravitational mass used in Newton's gravitational equation ?

eQ: What is the value of gravity at the center of the earth and why ?

6. A spherical cavity of radius 8 m has its centre 15 m below the surface. If the cavity is full of water and is in rocks of density 2.4 Mg/m^3 , what is the maximum size of the gravitational anomaly ?

Note, we are NOT calculating the absolute value of the force interaction between the Earth's mass and the mass anomaly associated with the water in the buried spherical cavity. We are just calculating the variation in the gravity field associated with replacing the rock in the spherical cavity with water. Thus, we can derive a simple approximation for the gravitational variation (Eqn. 8.5)

$\delta g = G \frac{\Delta m}{d^2}$ where Δm is the change in mass from the reference state (all rock) and d is the distance between the center of mass of the water filled cavity and the place where the gravity measurements is

made at the earth's surface. To calculate Δm , we note that the volume of a sphere is $V = 4/3 * \pi * r^3$ and hence the mass anomaly $\Delta m = V * \Delta \rho$ where $\Delta \rho$ is the density difference between the water and rock (1.40 Mg/m^3). Plugging in the number, we get

$$\delta g = G \frac{\Delta m}{d^2} = G \frac{V * \Delta \rho}{d^2} = 6.67e^{-8} \frac{2.14e^3 * 1.40}{15.0^2} = 8.9e^{-7} \text{ m/s}^2 \text{ or } 0.089 \text{ mGal}$$

7. Which one of the following is NOT true. The value of little-g varies over the surface of the earth:

- (i) True. If density, hence mass distribution varies, gravity will vary.
- (ii) True. The ellipsoidal figure of the earth and topography makes variable gravity.
- (iii) False. All other things equal, there is NO systematic variation of gravity with longitude.
- (iv) True. The gravity field systematically varies due to two effects: 1) the earth is NOT a sphere, but a flattened ellipsoid, i.e., the equatorial radius is 40 km larger than the polar radius; 2) the earth is a non-inertial reference frame due to its daily rotation with respect to the stars. Any object (e.g., mass in gravimeter) on this rotating body will experience the non-inertial reference frame centrifugal force directed away from the rotational axis. If an object is in-motion with respect to the planet, it experiences the non-inertial reference frame Coriolis force.
- (v) Same answer as (iv). One's distance from the poles (north or south) is just the definition of latitude.

8. How does little-g vary between the surface ($r_1=6400 \text{ km}$) and one kilometer up ($r_2=6401 \text{ km}$) ?

Let us subtract the two little-g acceleration values at distance from the planets center of mass for radius r_1 and r_2 . Let us ignore the vector property of gravity and do a 1-dimensional problem whose only independent variable is radius (r).

$$g(r) = G \frac{M_e}{r^2} \quad \text{acceleration} \left(\frac{m}{s^2} \right)$$

$$\Delta g = g(r_1) - g(r_2) = 6.672e^{-8} * 5.97e^{21} \left(\frac{1}{(6400e^3)^2} - \frac{1}{(6401e^3)^2} \right)$$

$$\Delta g = 3.983e^{14} * 7.627e^{-18} = 0.003 \text{ m/s}^2 = 300 \text{ mGal}$$

This is an inaccurate calculation because we need to use more than four significant figures given the huge variations in the different term exponents (10^{14} and 10^{-18}). A more accurate calculation is provided simply by using the free air gravity correction formula.

$$\Delta g(z) = 0.3086 \text{ mGal/m} * \text{height (m)} = 0.3086 * 1e^3 \text{ m} = 308.6 \text{ mGal}$$

This positive value is the number added to the measured gravity to CORRECT for the 1 km increase above the earth's surface. So, the measured gravity at an altitude of 1 km would be 308 Mgal less than as measured at the earth's surface.

9. If you took a gravimeter 1 km down a mine in rocks of density 2.3 Mg/m^3 , the gravity would change by how much ?

There are two gravitational effects (changes) to add together: the free air effect due to moving closer to the earth's center of mass and the Bouguer effect of the mass above the gravimeter whose mass would pull upwards. The tricky part of this problem is getting the sign correct and remembering that we are asking for what the gravitational 'effect' is not what the gravitational 'correction' is.

The magnitude of the free air effect is provided in Question 8 (309 mGal round off). The sign of this effect will be positive (increase gravity).

The Bouguer gravity effect is given by the Bouguer formula:

$$\Delta g_b(\Delta\rho, t) = 2\pi G \Delta\rho t \text{ mGal} = 2\pi * G * (2.3 - 0.0) * 1e^3 = 96 \text{ mGal}$$

Because the mass is above the gravimeter which is 1 km down a well, the mass pulls the gravimeter-mass upwards, opposite to the earth's gravity pull which is down. Therefore, the Bouguer anomaly should be negative.

Adding the two effects, the total change in gravity measured by the down-well gravimeter is +309 + (-96) mGal which equals a +213 mGal INCREASE in the measured gravity.

- 10. A person having carried out a microgravity survey to locate a lost shaft (filled with air), creates a gravity profile that shows a small gravitational dip in it (i.e., the measured gravity goes down or gets less and the anomaly has a negative sign). The surveyor notes that the dip in the gravity coincides with a dip (depression) in the otherwise level ground surface and thus she says she has located the mine shaft. But, then you find out that she has not corrected her data for topography (both free air and Bouguer effects). Discuss whether applying the topography corrections might result in the gravity dip disappearing.**

The combined free air and Bouguer gravity equation 8.11 shows that free-air gravitational effects (movement above or below one's datum) will dominate the equation and determine its sign. Thus, using the flat surface as a datum, the Bouguer gravity correction will have a negative value because the topographic dip is below the flat surface datum. This could make the measured dip in the gravity field that is coincident with the topographic dip get bigger.

Said another way, when one corrects for the fact that the gravity measured in the topographic dip will be higher because the measurement point is closer to the earth's center of mass, then the free-air correction (0.386 mGal/m) will be subtracted from the measured data. This will make the Bouguer anomaly even more negative.

- 11. The mean radius of the Earth is 6371 km. On taking a gravimeter 1 km above the earth's surface in a balloon, you would expect the value of little-g to decrease by how much ?**

Form a ratio of the gravity at r_e (6371 km) and r_e+1 and then turn the ratio into a percent by multiplying by 100.

$$g(r_1) = G \frac{M_e}{r_1^2}, \quad g(r_2) = G \frac{M_e}{r_2^2}, \quad \text{ratio: } \frac{g(r_1)}{g(r_2)} = G \frac{M_e}{r_1^2} / G \frac{M_e}{r_2^2} = \frac{r_2^2}{r_1^2} = \frac{6371^2}{6372^2} = 0.99968$$

To convert the ratio to percent change, multiple by 100. Thus, the gravity at 1 km height is 99.97% of its values measured at the surface. So, the gravity has decrease by 0.03% (100.0 - 99.97)%.

- 12. The International Gravity formula describes gravity:**

- (i) Only at the sea surface. Wrong. The IGF describes gravity only with respect to latitude and does not care whether one is over sea or land.
- (ii) On a surface simplified to be a sphere approximating the earth. Wrong. The IGF uses the best fitting ellipsoid for the earth's figure.
- (iii) To **ONLY** allow for the equatorial bulge. Wrong. It allows for the bulge and centrifugal forces.
- (iv) To allow for the centrifugal forces associated with the earth's rotation. True.
- (v) To allow for both the earth's rotation and the equatorial bulge. Exactly what the IGF accounts for.

13. An ancient burial chamber is to be found using a microgravity survey. The chamber is about 4 m across and is covered by about 3 m of material of density 2 Mg/m³. Estimate: (a) the maximum magnitude of the anomaly; (b) a suitable grid spacing assuming the total error in measurement is 0.1 mGal.

Assume the burial chamber is a sphere of radius 2 m with the sphere's center at its burial depth (3 m) plus the sphere's radius (2 m) to give a spherical center depth of 5 m. Use the gravity formula (Eqn. 8.5) that gives the maximum gravity value when measured directly over the center of a sphere.

$$\delta g = G \frac{\Delta m}{d^2} = G \frac{V * \Delta \rho}{d^2} = G * (4/3) * \pi * r^3 * \Delta \rho / d^2 = \frac{64}{75} * \pi * G$$

$$\delta g = 1.8e^{-7} m / s^2 = 0.02 mGal = 20 \mu Gal$$

The relation between the depth of the center of a spherical mass anomaly and the gravitational half-width is: $d = 1.3 * \text{half-width (m)}$. Therefore, the half-width is equal to $d/1.3$ which for $d=5$ m is about 4 meters. Thus, if one desires to measure the gravity anomaly associated with the burial chamber, one should sample at least every meter so that the gravity anomaly is not missed.

14. Describe how you would carry out a microgravity survey to determine *the lateral position* of dense mine workings (density of 2.6 Mg/m³) that is buried beneath 20 m of rocks of density 2.1 Mg/m³. Assume the gravimeter is accurate to 5 μGal. Estimate the accuracy in height and position required for these errors to be less than the gravimeters accuracy.

Little information is given, which is intentional and a real-world situation. All we know is that there is some unknown volume of high density mine workings below 20 m. But, we do know that the mine working density is 2.6 Mg/m³ and the overlying rock's density is 2.1 Mg/m³. Thus, we know that we are seeking to find a positive gravity anomaly.

Inspection of the depth rules (Fig. 8.19 page 121) shows the gravity anomaly half-width relations for different mass anomaly geometries: sphere, cylinder, dipping sheet, irregular body. Most useful for this problem is figure (d) for an irregular body where the relation between the depth to the top of the mass anomaly and the peak and slope of the anomaly are:

$$d \leq 0.86 * \frac{\delta g_{\max}}{\left(\frac{dg}{dx}\right)_{\max}}$$

where d is maximum depth to top surface (20 m), δg_{\max} is the peak height (in

mGal) of the gravity anomaly, and $(dg/dx)_{\max}$ is the maximum slope of the gravity anomaly (mgals/m). Substituting $d=20$ m into the equation and rearranging gives:

$23.2 * \left(\frac{dg}{dx} \right)_{\max} \leq \delta g_{\max}$. Now, we know that the maximum gravity value δg_{\max} will be greater than about 23 times the maximum gravity slope (also known as the first derivative of the function g with respect to distance x).

We also know that the maximum gravity slope will be near the edges of the dense mines working mass. Therefore, if one can find the places where the gravity slope is a maximum, we can detect the approximate edges of the mine workings.

I would suggest that data be collected along a series of gravity measurement traverses that are guaranteed to cross the 'guessed' boundaries of the mine works. From these traverses, the approximate edges of the mine workings can be found.

The vertical accuracy can be assessed by using the combined free-air and Bouguer anomaly equation 8.11:

$$\delta g = h * (0.3086 - 0.0419 * \rho) \text{ mGal where } \rho \text{ is } 2.1 \text{ Mg / m}^3 .$$

Substituting and simplifying gives: $\delta g = h * 0.2206 \text{ mGal / meter}$.

So for a 5 μGal gravimeter accuracy, the gravimeter height will need to be accurate to

$$\frac{5 \mu\text{Gal}}{.2206e^3 \mu\text{Gal / m}} = 0.023 \text{ m or } 2.3 \text{ cm (about one inch)} .$$

The horizontal accuracy can be assessed by using equation 8.8 and noting that England has a latitude of about 50°N.

$$\delta g_{lat}(\lambda) = 0.812 \sin(2\lambda) (\text{mGal / km} - \text{polewards}) = 0.800 \text{ mGal / km} - \text{poleward} .$$

So for a 5 μGal gravimeter accuracy, the gravimeter N-S position will need to be accurate to

$$\frac{5 \mu\text{Gal}}{.800e^3 \mu\text{Gal / km}} = 0.00625 \text{ km or } 6.25 \text{ m} .$$

15. Calculate the average density of the Earth, given little-g is 9.81 m/s² and the earth's mean radius is 6371 km.

Density is defined as mass divide by volume. Assuming a mean radius of the earth's sphere as 6371 km, the volume is:

$$V = 4/3 * \pi * r^3 = 1.083e^{12} \text{ km}^3 \text{ or } 1.083e^{21} \text{ m}^3 .$$

The total mass of the earth is found using equation 8.4

$$g(r) = G \frac{M_e}{r^2} \quad \text{acceleration} \left(\frac{m}{s^2} \right) .$$

Rearranging the equation to solve for M_e gives:

$$M_e (kg) = \frac{g(r = 6371km) * r^2}{G} = \frac{9.81(m / s^2) * (6371e^3(m))^2}{6.672e^{-8}(m^3 / Mg - s^2)} = 5.968e^{21} Mg$$

Therefore, the mean earths density is

$$\rho = \frac{5.968e^{21}}{1.083e^{21}} = 5.512 Mg / m^3$$

which is about twice the density value of granite. This means the deep earth must be much denser than the surface rocks with granite-like densities.

- 16. Explain why portions (portions means mass) weighed on the high Tibetan plateau (>4 km mean elevation) would be larger (greater mass) than portions weighted at sea level, if the portions are weighted using a spring balance ? But, the portions would NOT have to be greater on the Tibetan plateau if a mass balance is used.**

The gravity on the Tibetan plateau will be less with respect to sea-level because the free-air gravity decrease with altitude is bigger than the Bouguer gravity field effect due to the extra mass associated with the topography (see equation 8.11). Thus, to get the same weight of 'portions' measured on the Plateau and at sea level, the 'portions' (mass) weighed on the Plateau would have to be larger than the 'portions' (mass) weighed at sea level.

eQ: What is the difference between an object's mass and weight (again) ?

- 17. The radius of the planet Mars is 3394 km and the mass is 0.108 (10.8%) of the mass of the earth. Calculate the value of little-g at the surface of Mars in MKS units.**

The mass of Mars is thus 0.108 * Mass-earth which equals 0.108* 5.97e²¹ Mg which equals 6.45e²⁰ Mg.

Therefore, the gravitational acceleration at Mars surface is:

$$g(r) = G \frac{M_e}{r^2} = 6.672e^{-8} * \frac{6.45e^{20}}{(3394e^3)^2} = 3.74 \left(\frac{m}{s^2} \right)$$

eQ: So if you exert an upward force with your leg muscles to jump into the Mars air, why would the height of your jump be different with respect to the earth ?

- 18. Compared to the Earth, the mass of the Moon is about 1/80 and its radius is a quarter. How does the surface gravity ratio compare between the Earth and the Moon ?**

$$\frac{M_m}{M_e} = \frac{1}{80} \quad \text{and} \quad \frac{R_m}{R_e} = \frac{1}{4}$$

So, the calculated gravity ratio is

$$\frac{g_m}{g_e} = \frac{G \frac{M_m}{r_m^2}}{G \frac{M_e}{r_e^2}} = \frac{M_m * r_e^2}{M_e * r_m^2} = \frac{1}{80} * \frac{4^2}{1^2} = 0.2 \text{ (no units)}$$

Convert to percentages by multiplying by 100 gives the Moon's gravity to be 20% of the value of the earth's. A laborious way to answer this question would be to calculate the Moon and Earth gravity separately and then ratio the values.