

Geothermics Chapter 17 HW Answers

General questions and answers

1. Describe the three ways heat energy can be moved.

Heat energy can be transported (moved from place A to B) via three processes: Heat conduction moves heat by diffusing the temperature gradients at the atomic scale, convection physically moves the heat by advection of parcels, and electromagnetic radiation moves heat as electromagnetic infrared frequency waves. The heat energy ALWAYS flows from the hot high-T regions into the cold low-T regions!

2. Describe the difference between heat and temperature.

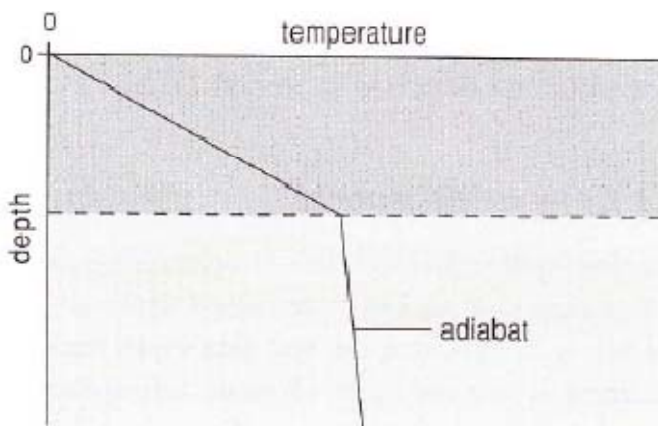
Heat is the most dispersed (entropic) form of energy. Energy can be defined as the ability to do work. For a gas, temperature is just the mean speeds of the molecules that 'fly' around; for a solid, temperature is best thought of as the vibrations of the crystal lattice of the atoms that make up the minerals. Thus, temperature is a measure of 'energy intensity'.

So, how does temperature relate the heat energy? Simple, every rock's specific thermal capacity (or heat capacity) has been measured by simply measuring how much heat energy must be added to a rock to raise its temperature 1°C.

Q (heat in Joules) = m * C * ΔT : m is mass, C is heat capacity, ΔT is change in temperature.

e.g., the higher the heat capacity of a rock, the more heat energy must be added to get a 1°K rise in the rocks temperature.

3. Define thermal gradient, geotherm and adiabat.



The **thermal gradient** is just the slope of the temperature vs. depth curve. If the temperature at the surface (depth=0 km) is 0°C and the temperature is 30° C at one km depth, then the thermal gradient is just $\Delta T/\Delta z = (30^\circ - 0^\circ) / 1 \text{ km} = 30^\circ \text{ C/km}$ or 0.03° C/m . The temperature profile of the earth is often called the **geotherm**.

An **adiabat** is a special temperature profile that results when a parcel has no heat flow across its boundary. Thus if the parcel is compressed into a smaller volume, with no change in the parcel's heat content, then the temperature follows the **adiabatic gradient**. The adiabatic gradient in the earth's mantle is about +0.5°C per km going downward (increasing pressure)

4. What is heat conduction?

Heat conduction occurs whenever the temperature gradient is non-zero. The heat always flows from the hot regions into the cold regions. This heat flow via conduction occurs at the atomic scale by the vibrational crystal lattice energy in the hot regions diffusing into the atoms in the cold regions. What drives

this process? The second law of thermodynamics which wants to drive all systems to equilibrium where the temperature is the same (constant) everywhere and hence there are no thermal gradients. Equilibrium is the state where the energy is the most dispersed which means the entropy is maximal. The heat conduction equation (Eqn. 17.2) is:

$$q \left(\frac{W}{m^2} \right) = \frac{\Delta T \left(\frac{^{\circ}C}{m} \right)}{\Delta z \left(\frac{m}{m} \right)} * K \left(\frac{W}{m - ^{\circ}C} \right).$$

5. What is heat production?

Nuclear decay occurs when the nucleus 'ejects' either an alpha (He nucleus) particle or an electron particle or a gamma electromagnetic wave (called ray often). These particles come out of the nucleus at very high speeds (0.1% of the speed of light) and hence have huge kinetic energy that gets transformed into heat.

Present day heat production from radioactive decay in the Earth is produced mainly by nuclear decay of the isotopes ^{238}U , ^{235}U , ^{232}Th and ^{40}K and has a value of $6.18 \times 10^{-12} \text{ W/kg}$ (Turcotte and Schubert, 1982).

Today, about half the 46 TerraWatts (10^{12} W) of heat flux across the earth's surface comes from radioactive heat production in the earth's crust and mantle. The other half of the planetary heat flux comes from the heat left over from when the earth was created 4.5 Ga years ago.

An important observation to know in interpreting the earth's heat flux, it the fact that radioactive elements are highly concentrated in the continental crust, whereas, the oceanic crust is highly depleted in radioactive elements. This is reflected in Fig. 17.4 by the oceanic heat flux being constant with depth, whereas the continental heat flux increases towards the surface.

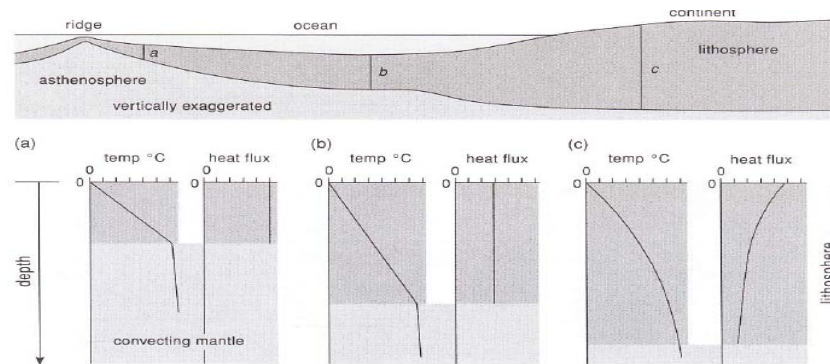


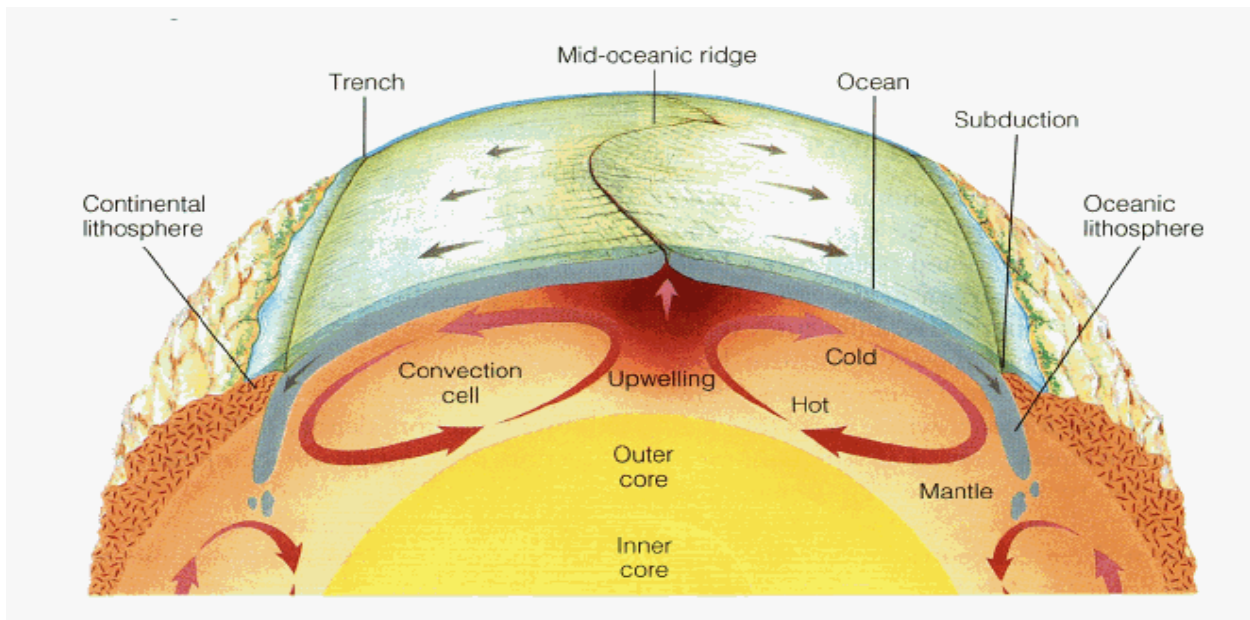
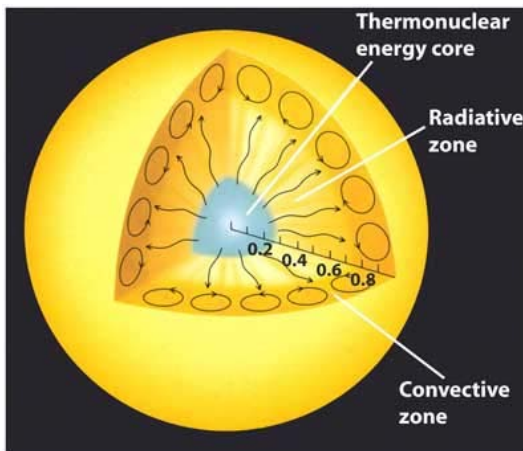
Figure 17.4 Temperatures and heat fluxes through oceanic and continental lithosphere.

6. What is convection and how does convection transport (move) heat?

Convection is everywhere in the oceans, atmosphere, solid earth, Sun, tea-pot, etc. Convection is a phenomena that occurs when the deep rocks can flow (solid state creep chap. 9) driven by buoyancy forces (Archimedes Principle chap. 9). Buoyancy forces arise in the earth's mantle because hot regions are less dense (under mid ocean ridges) and cold regions (subducting slabs) are more dense. This change in density due to changes in temperature, when multiply by little-g, produce the forces that 'drive' convection. Thus, convection is a phenomena that DOMINATELY moves (transports) heat NOT by conduction (although conduction always occurs), but by moving hot regions upwards and cold regions downward. The net effects is the transport (move) the earth's heat to the near surface where it is conducted through the lithosphere

and finally into the atmosphere. Just like conduction that moves heat from hot (high-T) to colder (lower-T) regions, convection moves (transports) heat from the high-T (3500°C) core mantle boundary to the low-T surface (11°C).

So, what determines whether heat is transported by convection? Convection occurs when the strength (viscosity) of the mantle is low enough that the buoyancy forces can drive flow at geologic rates of 1-cm/yr (10 km/Ma). This is all very well studied. Whether the earth was convecting heat or not WAS the big question until the plate tectonics revolution in the 1960's. Note that the gravity and isostasy measurements were very important because they suggest that the earth's asthenosphere could flow on Ma time-scales!



Chapter HW problem answers

1. Distinguish between free and forced thermal convection and conduction and give examples.

Thermal conduction, as discussed above, is always operative wherever there is a thermal gradient (temperature difference) between two regions. The heat conducts at the atomic scale as the vibrational energy of the solid crystal lattice diffuses outward in time from the hot to the cold region! Note that there is no movement (flow) of the material, it is static.

Thermal convection occurs when the viscosity of a substance (i.e., mantle) is low enough that the buoyancy forces formed by the temperature gradient between the top and bottom are great can 'drive' significant flow rates (e.g., 1-10 cm/yr in mantle). Noteworthy is that with respect to the gravity field the top must be cold and the bottom hot so that hot material is driven upwards and cold material is driven downwards. This is also called free convection because it occurs without any forcing other than the temperature gradient. Examples are thunderclouds, earth mantle convection.

Forced convection occurs when heat is physical moved by other forces (i.e., the fan that forces heat through the heating ducts of a house).

2. Why are two quantities measured to deduce the heat flux. Explain why this measurement does not include any convected heat.

In general, the earth's heat flow is so small (10-100 mW scale), that it would be hard to directly measure: e.g., an extremely well-insulated cup of water sitting on a square meter of surface rock would require months of heat flow to warm up much. Therefore, an indirect way to measure heat flow is to measure the two parameters on the right hand side of the heat flow equation $q=k*(dT/dz)$: that is the thermal gradient (e.g., in a borehole) and the thermal conductivity of the rocks in the borehole.

We certainly hope that a conduction measurement in a borehole contains **NO convected heat** as this would make the conduction measurement **wrong**. Convected (or advected) heat can be a problem due to ground water flow into the borehole. Also, the drilling and mud flow in the borehole can change the temperature gradient. One likes to let a borehole sit capped for a while before measuring the temperature gradients so that thermal equilibrium (or steady-state) is achieved.

3. Temperature corresponds to which of the following electrical quantities: (1) current. (2) voltage. (3) potential. (4) resistivity. (5) charge.

Advanced question. Ohm's law says: Voltage = Current * resistivity. Current is the flux of electron down a wire, which would be like heat flow. Voltage and potential are the same thing and represent a forcing that drives current-flow. So, voltage and temperature are like temperature in that temperature gradients also drive flow (heat flow). There is no equivalent to electrical charge in heat physics.

4. Sketch geotherms through (a) oceanic and (b) continental lithosphere. Why do they differ.

See Figure 17.4. They are different because continental crust has a factor of 20 higher heat production with respect to oceanic crust. This enrichment of continental crust in heat producing elements (e.g., Uranium) is caused by the different melting processes that make continental crust. Important: with significant heat production in the crust, the continental geotherm is no longer a straight line (like the oceanic crust/lithosphere geotherm).

5. Show by means of sketches the effect upon the temperature profile of (a) a rapid deposition of sediments, (b) a rapid overthrusting.

See Figure 10.9 and 10.10. The important point is to KNOW the difference between a steady-state (equilibrium) geotherm and a disequilibrium geotherm. A **steady-state geotherm does NOT change** in

time whereas a **disequilibrium geotherm DOES change in time** (for millions of years in the earth) until it reaches steady-state. So, if one starts with a steady state geotherm, and then either (a) rapidly (<1-2 Ma) deposits sediments at the surface or water temperature, or (b) thrust a large section of the upper crust over itself, a disequilibrium is created. This disequilibrium will then take 10-100 Ma to reach a new steady-state equilibrium geotherm.

6. The global heat flux at the earth's surface is large and exceeds global human energy use. Explain why it would be impossible to trap and use much of this heat energy.

The global heat flux through the large surface area of the earth ($4\pi \text{radius}^2$) is a large heat flux (45 TerraWatts or TW): e.g., total total energy use per unit time (power in Watts) by humans is about 18 TW. BUT, you would need to be a heat collector over the entire surface of the planet to capture and use the heat to drive steam turbines to make electricity.

7. Explain why geothermal energy is NOT a true renewable energy source.

Heat conduction in rocks is very very slow: i.e., rocks are actually very good heat insulators: e.g., brick and cement houses. Thus, geothermal energy can be extracted much quicker than the earth's heat flux can replace the extracted heat energy.

8. The heat flow of roughly what area of average continental surface (in km² units) would be required to match the power output of a 2000 MW (mega-watt) power station.

Watch your units!

Power (W) = Area (m²) * Heat-flux (W/m²) and 1 MW = 10⁶ W and 2000 MW = 2e⁹ W

Set Area=A and note that continental heat flux is about: $q_c = 5e^{-2} \text{ W/m}^2$

Calculate: $2e^9 \text{ (W)} = A \text{ (m}^2\text{)} * 5e^{-2} \text{ W/m}^2 \gg \gg A \text{ (m}^2\text{)} = 4e^{10} \text{ (m}^2\text{)} \text{ or } 4e^4 \text{ (km}^2\text{)}.$

Answer: 40,000 square kilometers.

9. Explain why caves ARE cool on hot days but warm on cold ones.

The heat capacity of rocks is very large. A large heat capacity means it takes larger amounts of heat (Joules) to produce a small temperature rise (°C): $Q = \text{heat-capacity} * \text{temperature}$. This large heat capacity of rocks means that variations in the atmospheric temperature outside the caves are thermally-buffered by the rocks large heat capacity. Hence, the temperature in the caves responds slowly to hot OR cold days.

10. Explain with sketches, how temperature varies with depth below ground due to daily and annual (seasonal) temperature variations.

See section 17.5 and Figure 17.3. Both daily and seasonally temperature variations can be approximated as a sinusoidal function : $\text{Temperature (time)} = \text{Amplitude} * \sin(w*t)$. The sinusoidal variations in temperature are damped by a negative exponential ($e^{-\alpha * \text{depth}}$) and the 1/e value of this negative exponential is called the **skin-depth**. For daily sinusoidal temperature fluxuations, this means

the skin depth to which the temperature fluxuations penetrate is only a few tens of centimeters! But, for yearly time scales, the temperature variations will have a much larger **skin-depth**.

11. Why does it take millions of year for the temperature profile in the Earth to return to equilibrium following a thermal disturbance to it.

Because the alumino-silicate rocks that make the earth have a very low thermal conductivity and thus it takes a long time (10-100 Ma) for thermal equilibrium to be attain after a thermal disturbance (e.g., overthrust, magma intrusion, sediment basin filling).

12. A thick layer of clay overlies granite. The thermal conductivity of the clay is half that of the granite. If the temperature gradient in the granite is 16°C/km, the thermal gradient in the clays would be? Assume thermal equilibrium and no heat production.

Important: Equilibrium and no heat production implies that the heat flux in the clay (q_1) is the same as the heat flux in the granite (q_2). Also, the book answer is wrong!

$$q_1 = k_1 * \left[\frac{dT}{dz} \right]_{clay} \quad q_2 = k_2 * \left[\frac{dT}{dz} \right]_{granite}$$

$$\text{Given : } q_1 = q_2, \quad k_1 = 0.5 * k_2, \quad \left[\frac{dT}{dz} \right]_{granite} = 16^\circ C / km.$$

$$\triangleright k_1 * \left[\frac{dT}{dz} \right]_{clay} = k_2 * \left[\frac{dT}{dz} \right]_{granite} \quad \triangleright 0.5k_2 * \left[\frac{dT}{dz} \right]_{clay} = k_2 * 16$$

$$\triangleright \left[\frac{dT}{dz} \right]_{clay} = \frac{16}{0.5} = 32^\circ C / km.$$